



Fuel Cell Reformer Control

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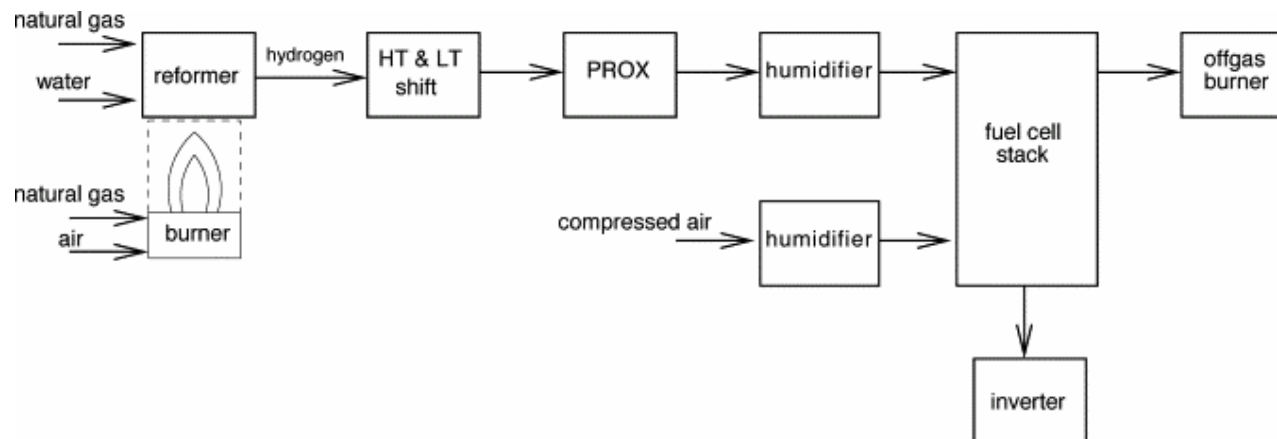


Presentation Outline

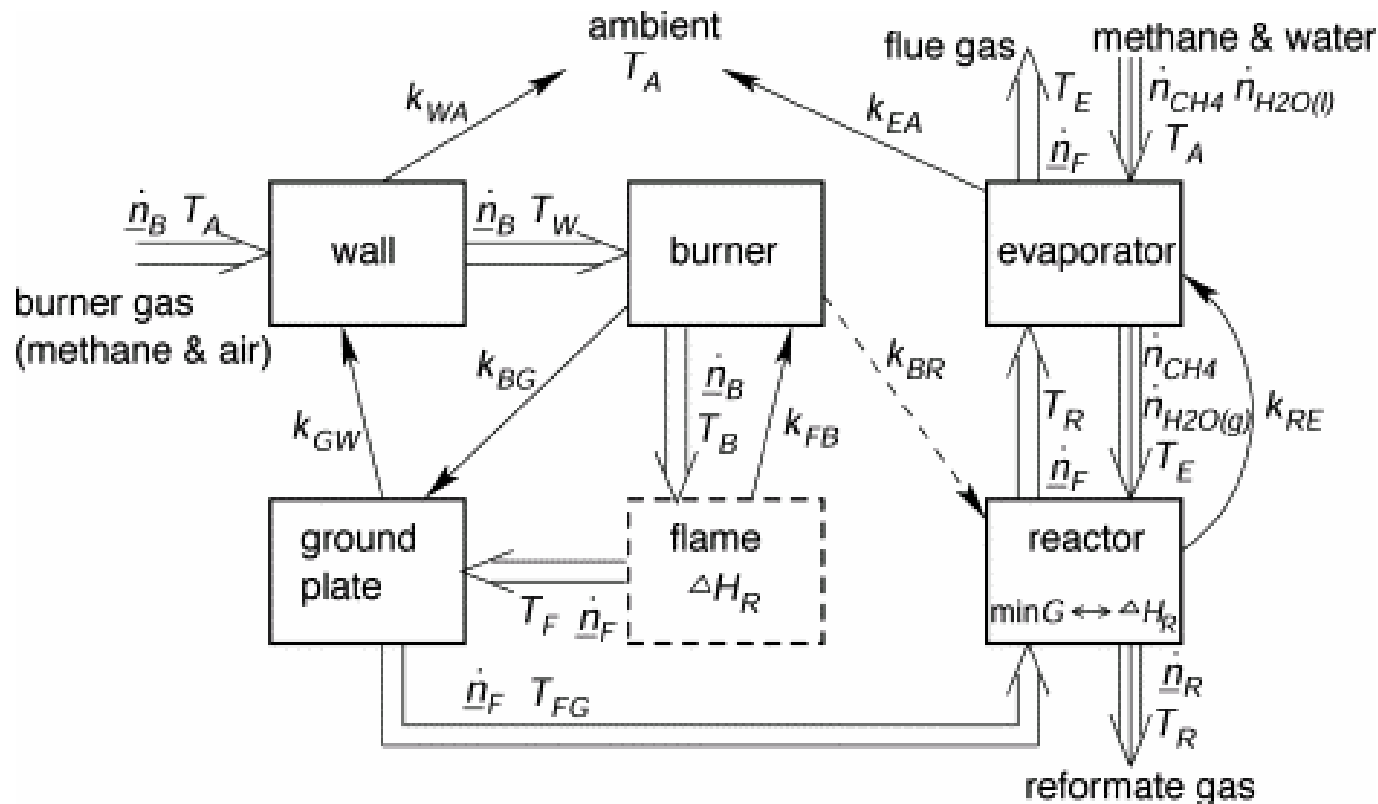
- Introduction
- Development of the state space model
- Modeling the system
- SISO control
- Multivariable control
- RGA analysis and pairing
- Disturbance rejection
- Directional sensitivity

Introduction

- Purpose: create final project
- Model
 - Steam reformer for residential fuel cell plant
 - From Jahn and Schroer, 2005



Development of State Space Model: Model Component Relationships



Single lines depict heat transfer (solid is conduction, dashed is radiation), double lines depict the burner gas flow, and triple lines depict the reformat gas flow.

Development of State Space Model: Dynamic Equations

$$C_W \frac{dT_W}{dt} = k_{GW} (T_G - T_W) - k_{WA} (T_W - T_A) - c_{p,B} \cdot \dot{n}_B (T_W - T_A)$$

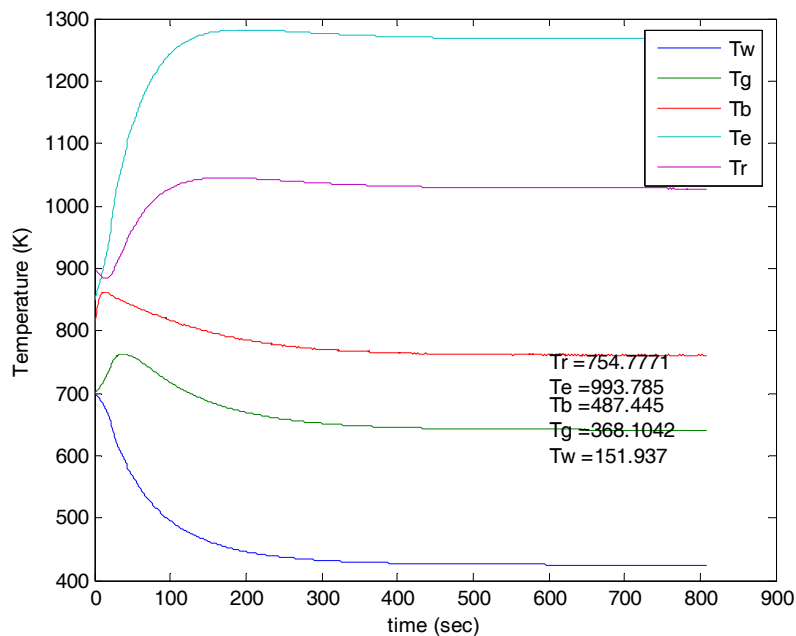
$$C_G \frac{dT_G}{dt} = k_{BG} (T_B - T_G) - k_{GW} (T_G - T_W) + k_{FG} \cdot c_{p,F} \cdot \dot{n}_F (T_F - T_G)$$

$$C_B \frac{dT_B}{dt} = k_{FB} (T_F - T_B) - c_{p,B} \cdot \dot{n}_B (T_B - T_W) - k_{BR} (T_B^4 - T_R^4) - k_{BG} (T_B - T_G)$$

$$C_E \frac{dT_E}{dt} = k_{RE} (T_R - T_E) + c_{p,F} \cdot \dot{n}_F (T_R - T_E) - r \cdot \dot{n}_{H_2O,i} - c_{p,H_2O} \cdot \dot{n}_{H_2O} (T_E - T_{H_2O}) \\ - c_{p,CH_4} \cdot \dot{n}_{CH_4} (T_E - T_{CH_4}) - k_{EA} (T_E - T_A)$$

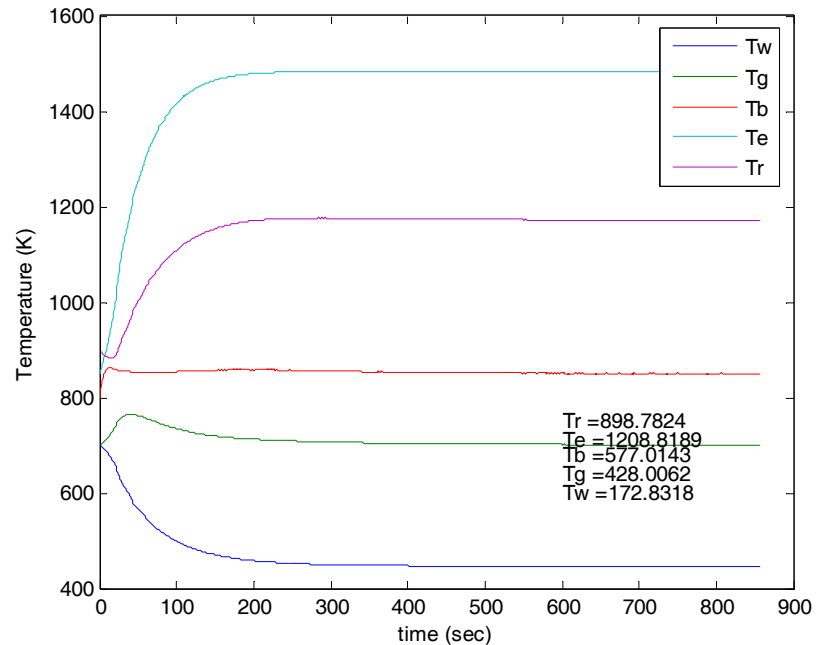
$$C_R \frac{dT_R}{dt} = k_{BR} (T_B^4 - T_R^4) - k_{RE} (T_R - T_E) + c_{p,F} \cdot \dot{n}_F (T_{FG} - T_R) \\ - \Delta h_0 \cdot \Delta \dot{n}_0 - \Delta h_1 \cdot \Delta \dot{n}_1 - c_{p,H_2O} \cdot \dot{n}_{H_2O} (T_R - T_E) - c_{p,CH_4} \cdot \dot{n}_{CH_4} (T_R - T_E)$$

Development of State Space Model: Changing nCH4i

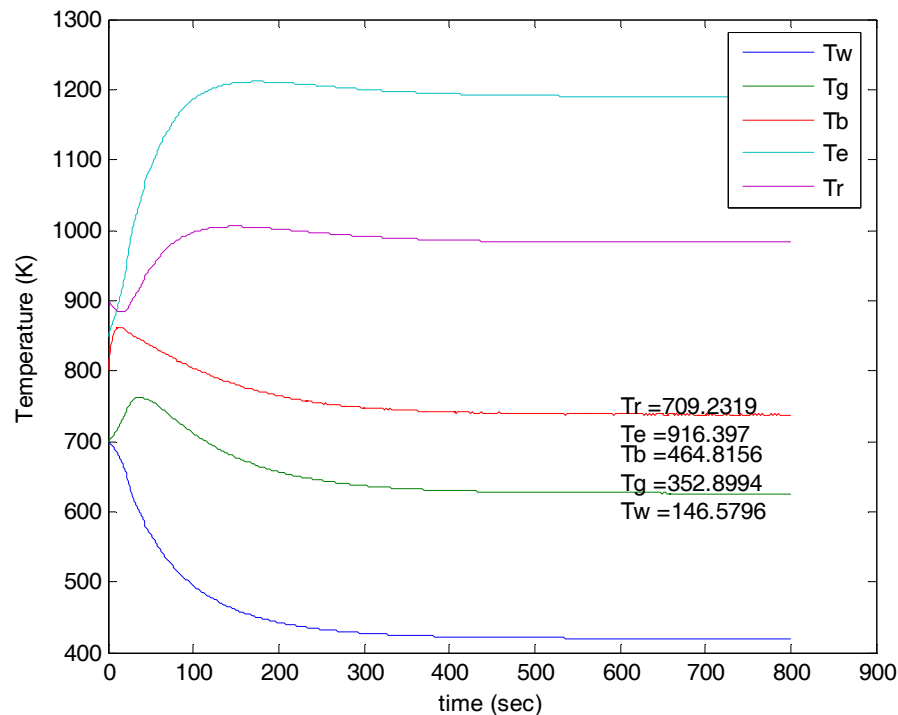


initial methane flow rate=10 SLPM
steam to carbon ratio=3.5
excess air ratio=5

initial methane flow=15 SLPM
steam to carbon ratio=3.5
excess air ratio=5

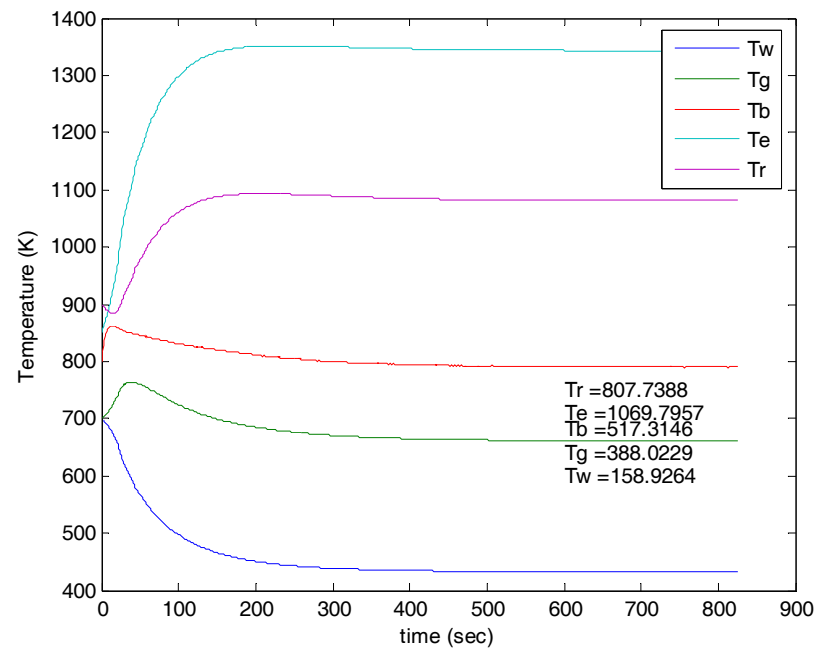


Development of State Space Model: Changing Steam to Carbon Ratio

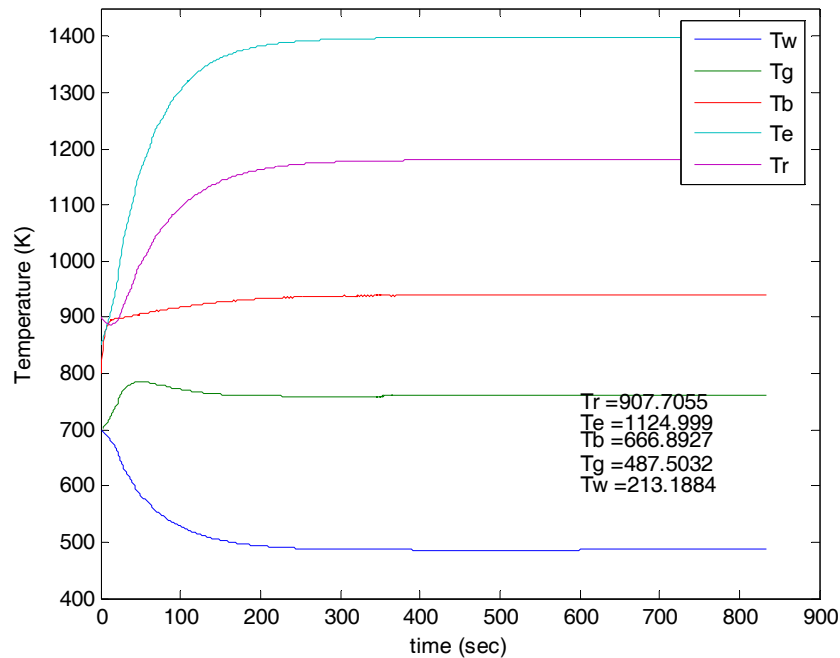


initial methane flow rate=10 SLPM
 steam to carbon ratio=3
 excess air ratio=5

initial methane flow rate=10 SLPM
 steam to carbon ratio=4
 excess air ratio=5

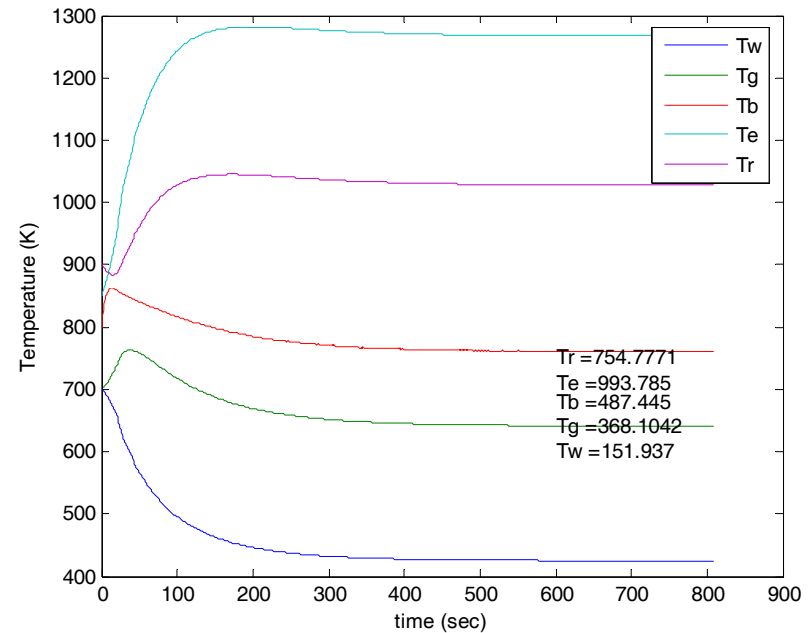


Development of State Space Model: Changing Excess Air Ratio



initial methane flow rate=10 SLPM
steam to carbon ratio=3.5
excess air ratio=4

initial methane flow rate=10 SLPM
steam to carbon ratio=3.5
excess air ratio=5

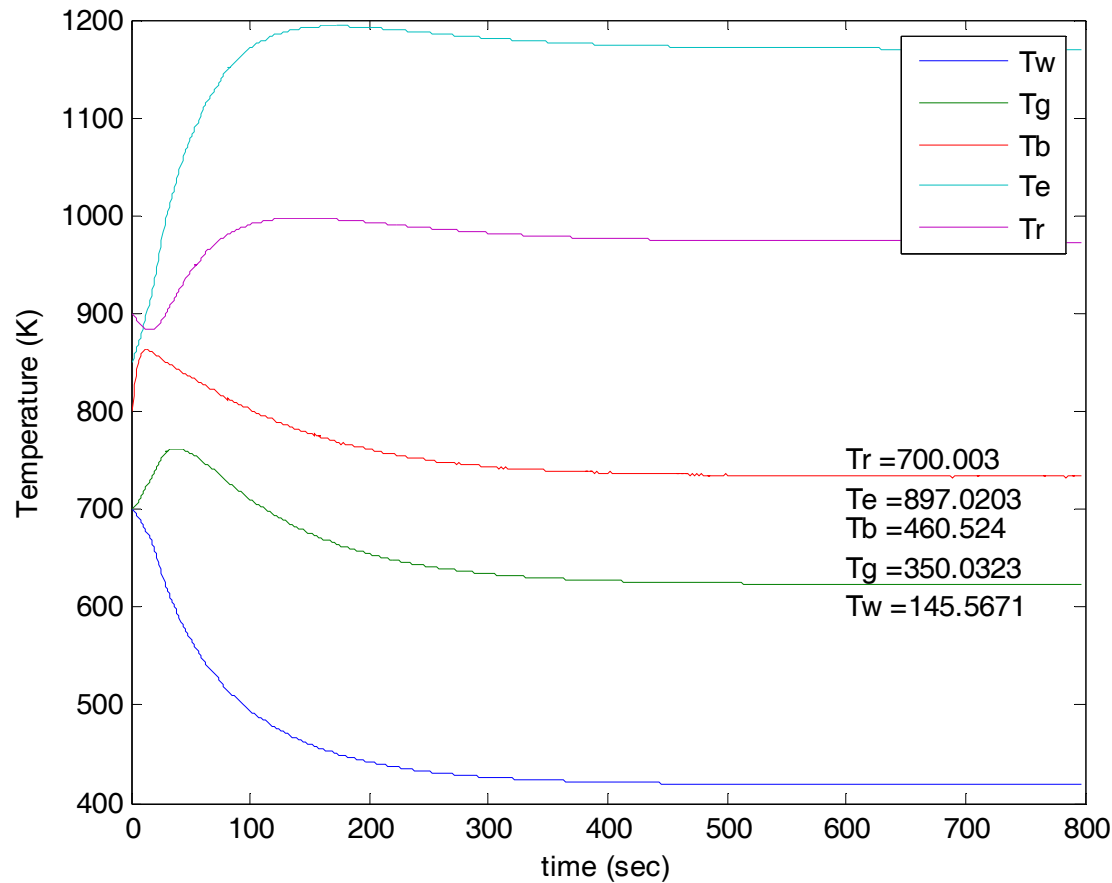




Development of State Space Model: Specified values

- Reformer temp = 700 deg C
- Steam to carbon ~ 3.5
- Methane flow rate ~ 10 SLPM

Development of State Space Model: Output Temperatures



Methane flow rate = 9.5 SLPM
Steam to carbon=3.0076
Excess air ratio=5

Development of State Space Model: States, Inputs, and Outputs

$$\text{States} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} T_W - T_{Ws} \\ T_G - T_{Gs} \\ T_B - T_{Bs} \\ T_E - T_{Es} \\ T_R - T_{Rs} \end{bmatrix}$$

$$\text{Inputs} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \text{excess air ratio } (\lambda) \\ nv \end{bmatrix}$$

$$\text{Outputs} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} T_B \\ T_R \end{bmatrix}$$



Development of State Space Model: Matrices

$$\dot{x}' = Ax' + Bu'$$

$$y' = Cx' + Du'$$

$$A_{ij} = \frac{\partial f_i}{\partial x_j}$$

$$B_{ij} = \frac{\partial f_i}{\partial u_j}$$

$$C_{ij} = \frac{\partial g_i}{\partial x_j}$$

$$D_{ij} = \frac{\partial g_i}{\partial u_j}$$



Development of State Space Model: Matrices

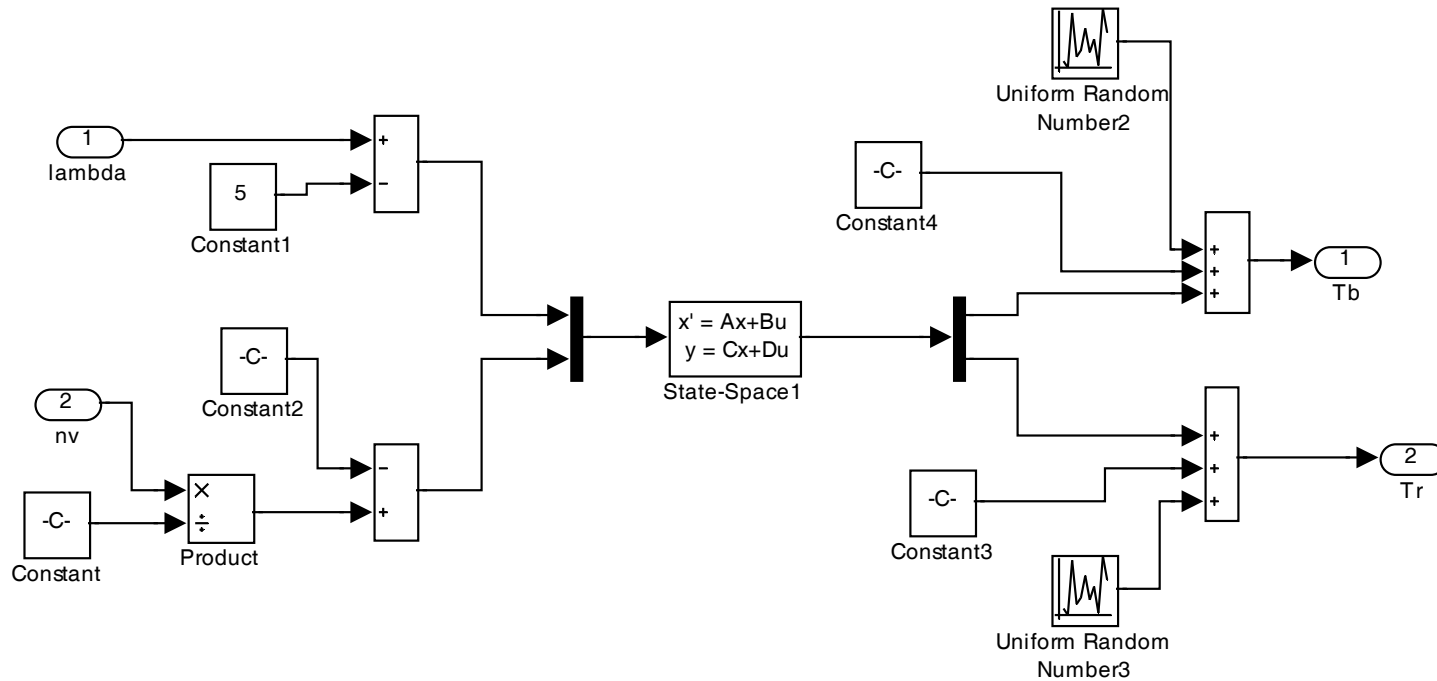
$$A = \begin{bmatrix} -0.001593 & 7.098 \times 10^{-4} & 0 & 0 & 0 \\ 0.002115 & -0.004911 & 0.0018443 & 0 & 0 \\ 0.034462 & 0.020455 & -0.058625 & 0 & -0.008232 \\ 0 & 0 & 0 & -0.00472 & 0.004436 \\ 0 & 0 & 1.4285 \times 10^{-4} & 0.004675 & -0.007322 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.02424 & -25.888 \\ 0.14687 & 156.0762 \\ -2.0898 & -2032.8766 \\ -0.05429 & -57.6894 \\ -0.075294 & -80.0119 \end{bmatrix}$$

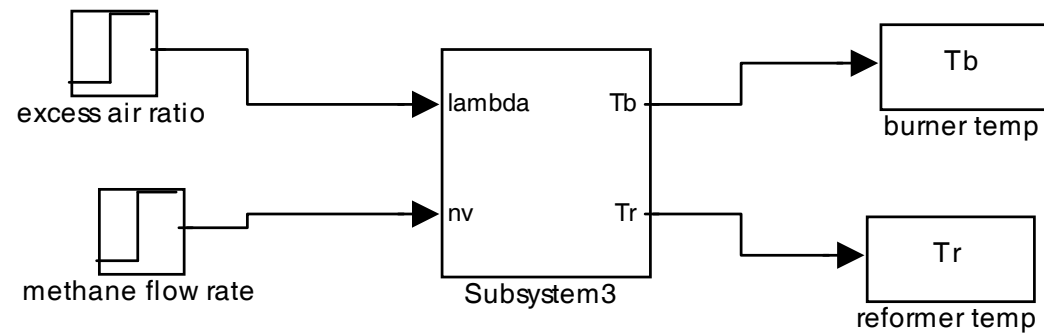
$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Development of State Space Model: Final Subsystem

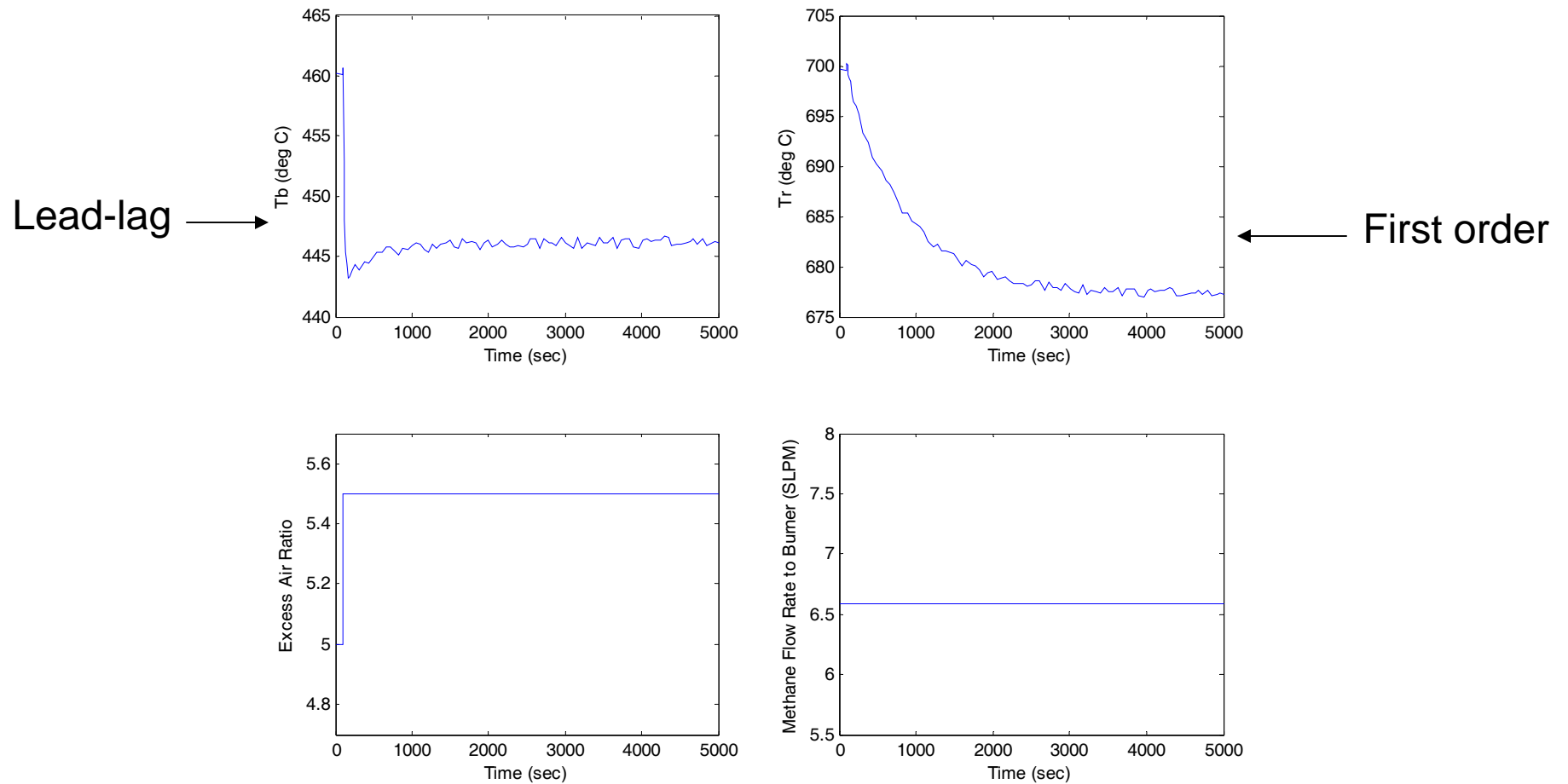


Development of State Space Model: Subsystem in Large System



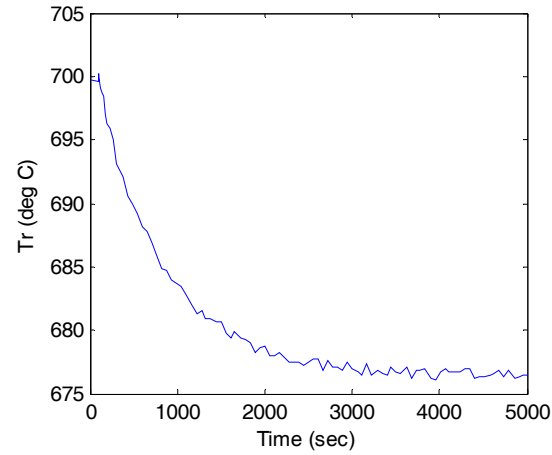
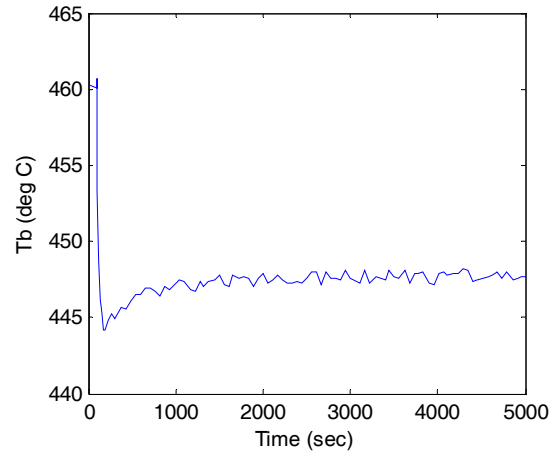
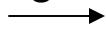
Model Development

Temperature responses to excess air ratio change of 0.5

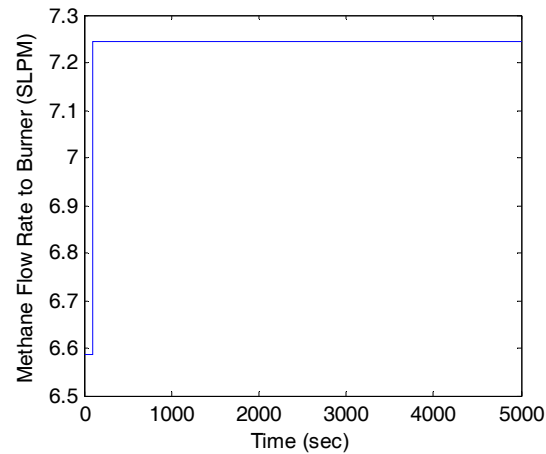
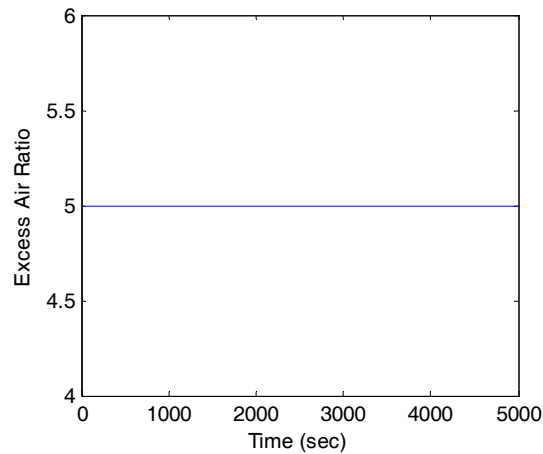
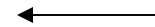


Temperature responses to a methane flow rate change of 0.5658 SLPM

Lead-lag



First order



Model Parameters

First order equation:

$$g_p = \frac{kp}{\tau_p s + 1}$$

$$kp = \frac{\Delta y}{\Delta u}$$

τ_p = time when 63.2% of change occurs

Lead-lag equation:

$$g_p = kp \left(\frac{\tau_n s + 1}{\tau_p s + 1} \right)$$

$$kp = \frac{\Delta y}{\Delta u}$$

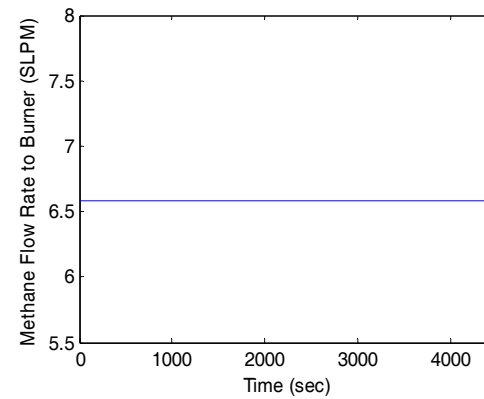
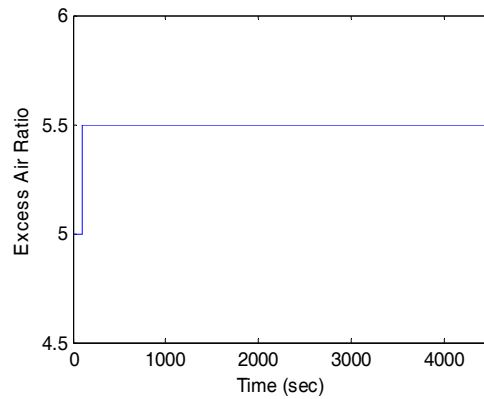
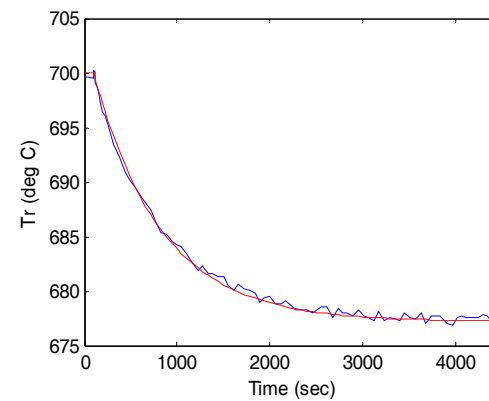
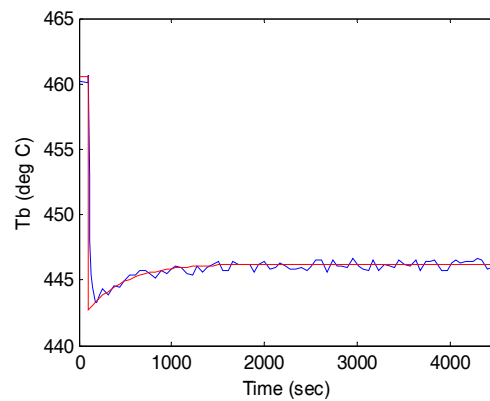
iterate to find τ_n, τ_p

Process Transfer Functions

$$\begin{bmatrix} -28.548^\circ\text{C} \frac{500 \text{ sec}(s)+1}{400 \text{ sec}(s)} & -19.39^\circ\text{C} \frac{520 \text{ sec}(s)+1}{400 \text{ sec}(s)+1} \\ \frac{-45.506^\circ\text{C}}{730 \text{ sec}(s)+1} & \frac{-35.69^\circ\text{C}}{710 \text{ sec}(s)+1} \end{bmatrix} \begin{bmatrix} u1 \\ u2 \end{bmatrix} = \begin{bmatrix} y1 \\ y2 \end{bmatrix}$$

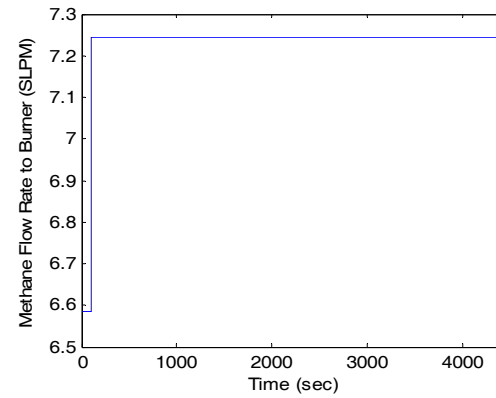
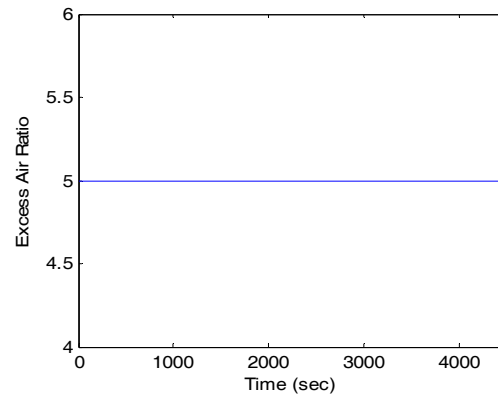
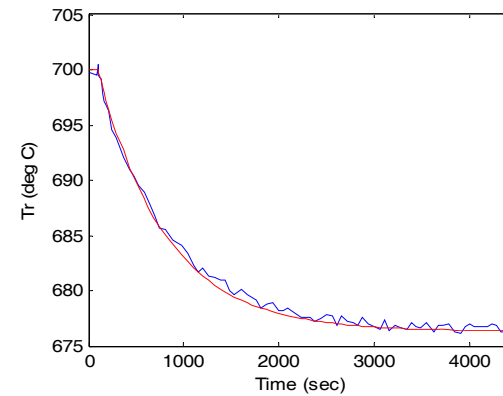
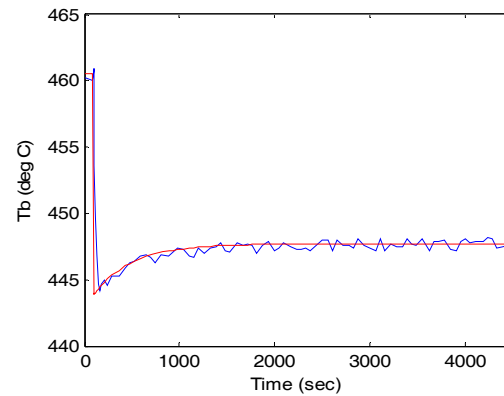
Process vs Model

Model and process responses to setpoint change in excess air ratio



Process vs Model

Model and process responses to setpoint change in methane flow rate





SISO Controller Development: IMC-based PID control strategy

PI controller w/ disturbance rejection
for first order transfer functions

$$g_c = kc \left(1 + \frac{1}{\tau_I s} \right)$$

$$kc = \frac{2\tau_p - \lambda}{kp\lambda}$$

$$\tau_I = \frac{2\tau_p\lambda - \lambda^2}{\tau_p}$$

PI controller w/ filter term for
lead-lag transfer functions

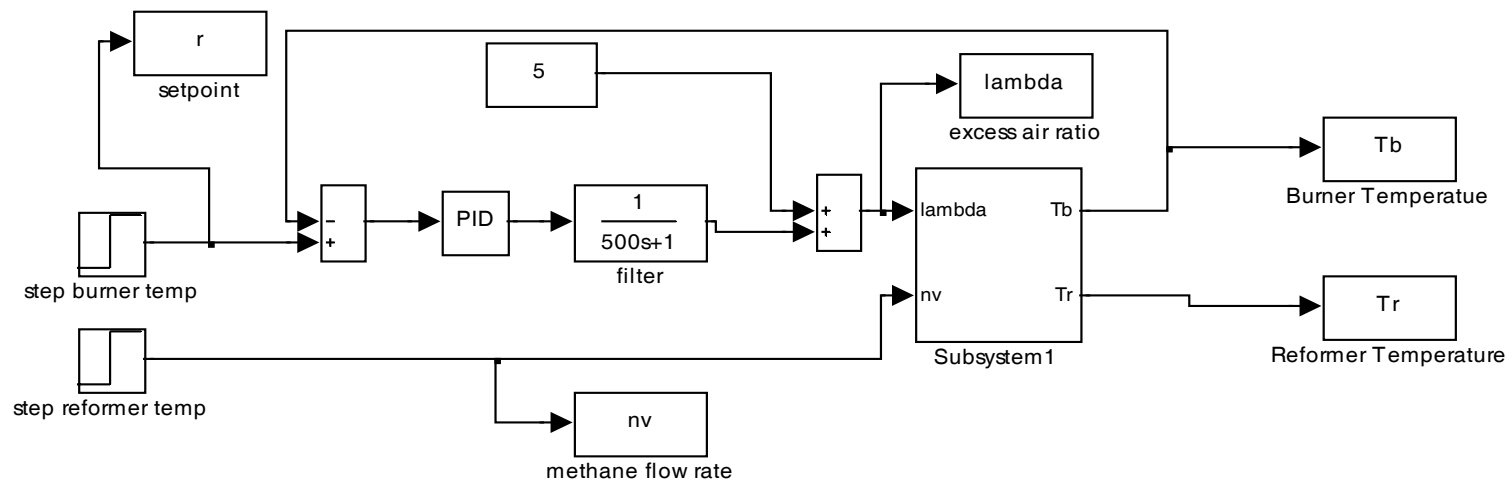
$$g_c = kc \left(1 + \frac{1}{\tau_I s} \right) \left(\frac{1}{\tau_F s + 1} \right)$$

$$kc = \frac{\tau_p}{kp\lambda}$$

$$\tau_I = \tau_p \quad \tau_F = \tau_n$$

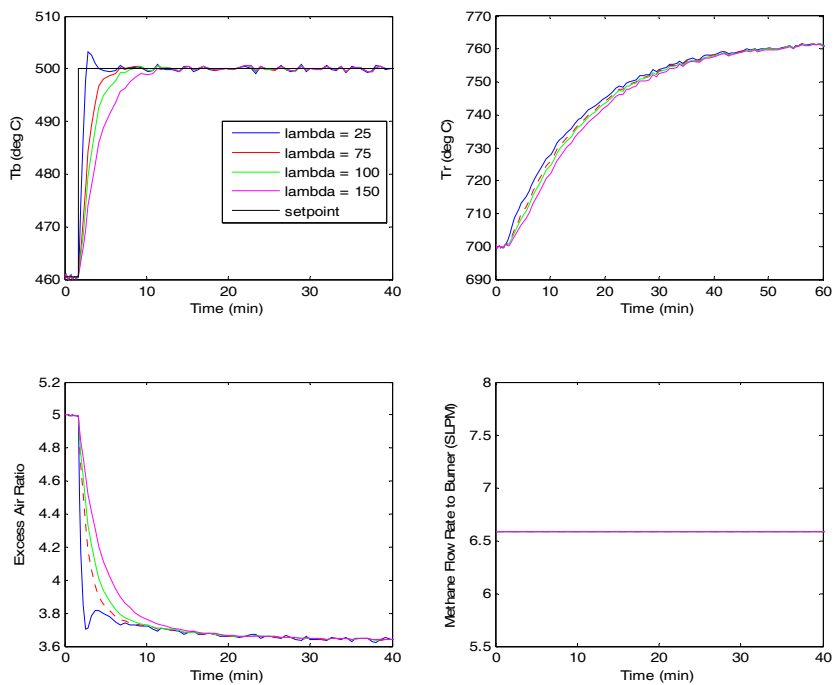
Simulink Diagram: SISO control

Burner temperature controlled by the excess air ratio

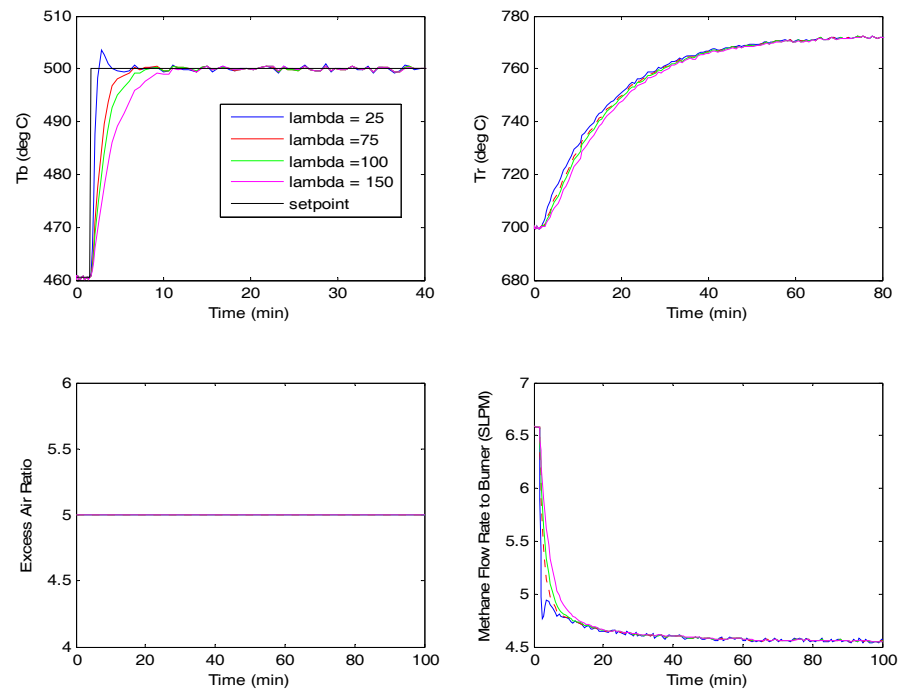


Burner Temperature Control

Control by excess air ratio

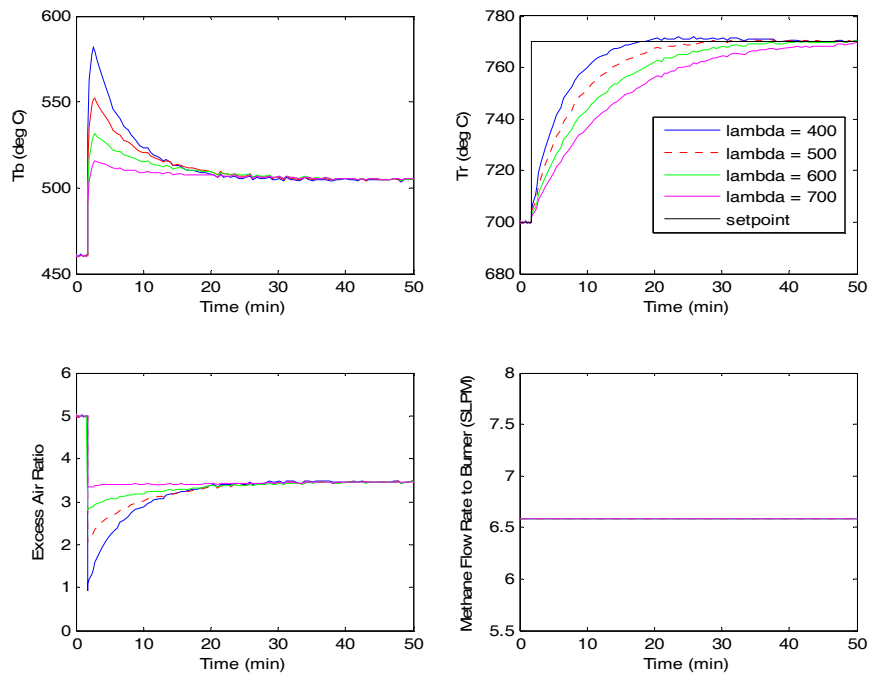


Control by methane flow rate

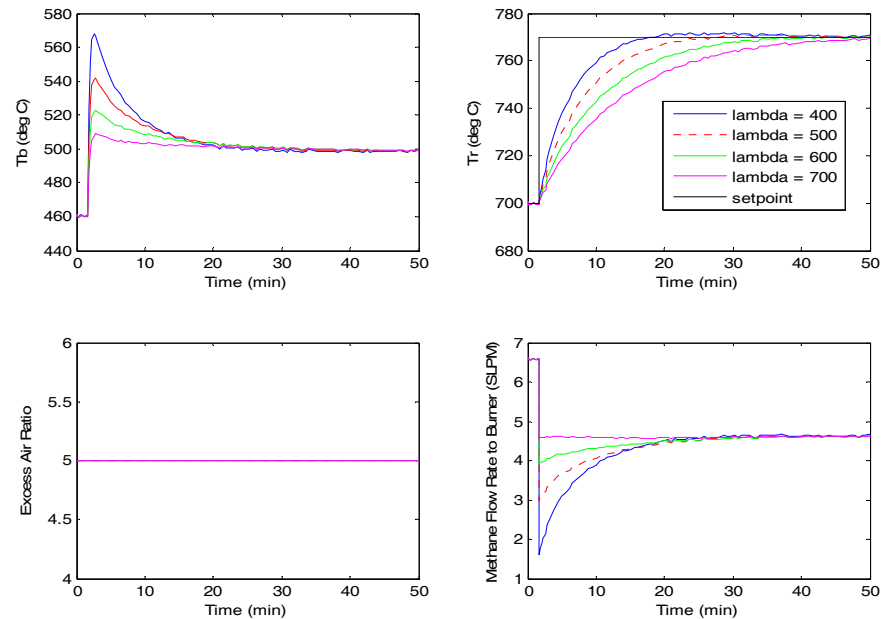


Reformer Temperature Control

Control by excess air ratio



Control by methane flow rate





SISO Controllers

y1-u1

$$g_{c11} = -0.1401 \left(1 + \frac{1}{400s} \right) \left(\frac{1}{500s + 1} \right)$$

y1-u2

$$g_{c12} = -0.2063 \left(1 + \frac{1}{400s} \right) \left(\frac{1}{520s + 1} \right)$$

y2-u1

$$g_{c21} = -0.0315 \left(1 + \frac{1}{706.85s} \right)$$

y2-u2

$$g_{c22} = -0.0383 \left(1 + \frac{1}{692.96s} \right)$$

Multivariable Control: RGA Analysis

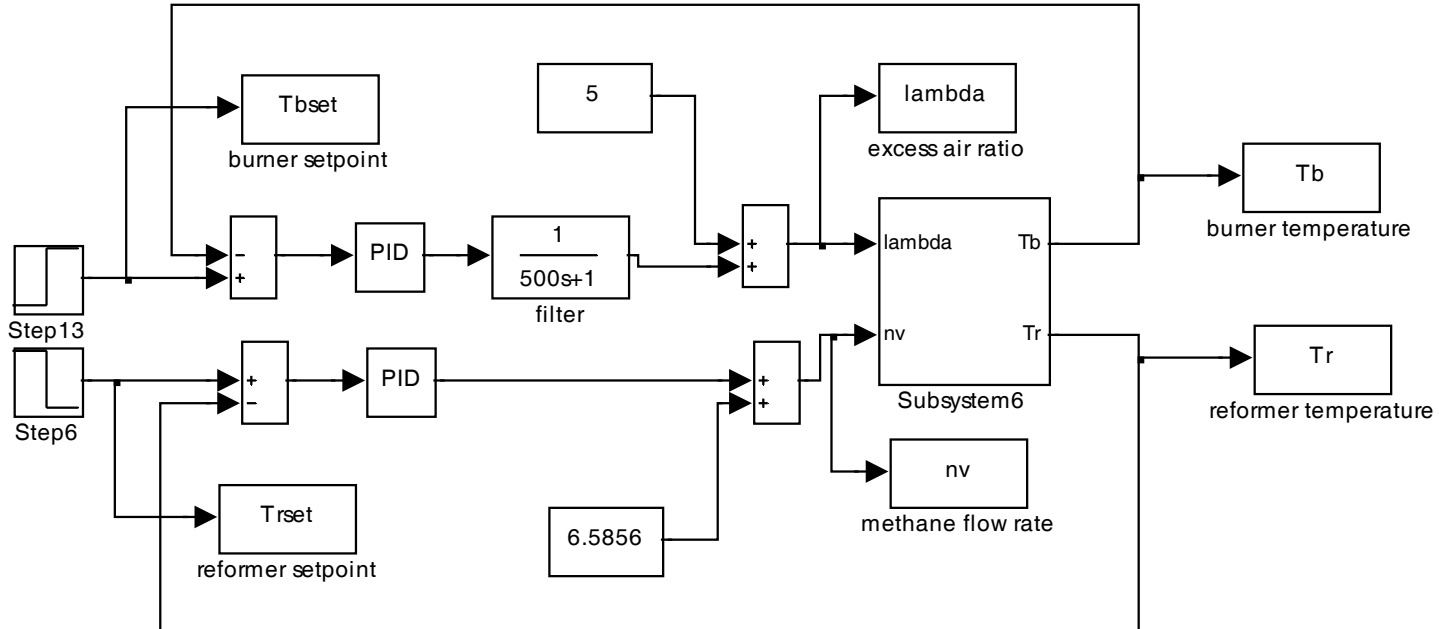
Process Gain Matrix

$$K = \begin{bmatrix} -28.548 & -19.39 \\ -45.506 & -35.69 \end{bmatrix} \longrightarrow \Lambda = \begin{bmatrix} \frac{k_{11}k_{22}}{k_{11}k_{22} - k_{12}k_{21}} & \frac{-k_{12}k_{21}}{k_{11}k_{22} - k_{12}k_{21}} \\ \frac{-k_{21}k_{12}}{k_{11}k_{22} - k_{12}k_{21}} & \frac{k_{11}k_{22}}{k_{11}k_{22} - k_{12}k_{21}} \end{bmatrix} \longrightarrow \Lambda = \begin{bmatrix} 7.463 & -6.463 \\ -6.463 & 7.463 \end{bmatrix}$$

Relative Gain Array

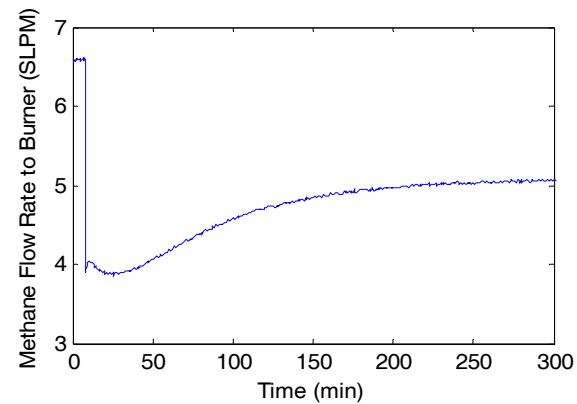
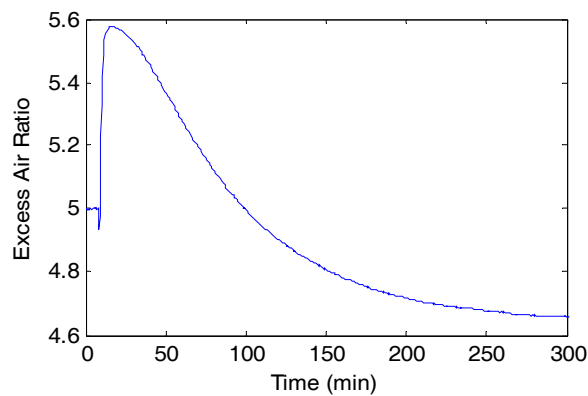
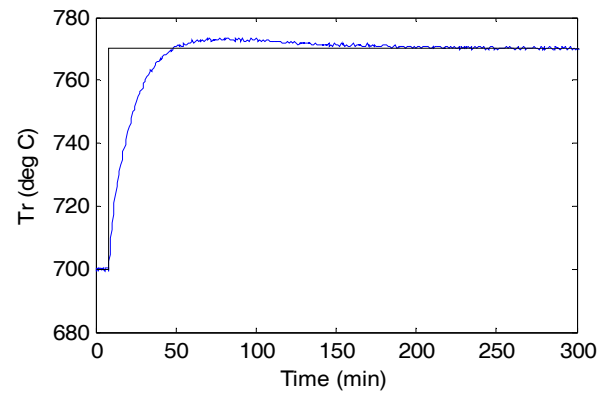
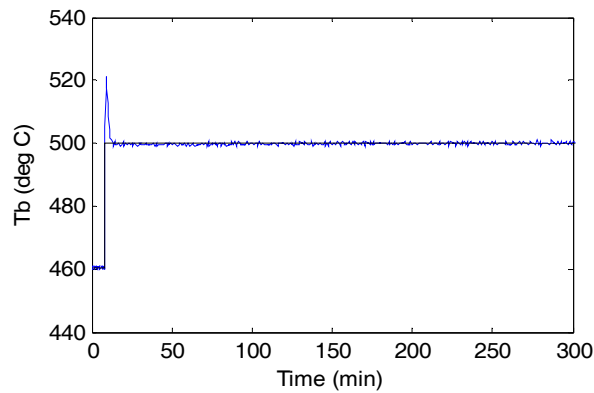
Do not pair on negative relative gain \rightarrow y1-u1 and y2-u2 pairings

Simulink Diagram: y1-u1, y2-u2 pairing



Multivariable Control:

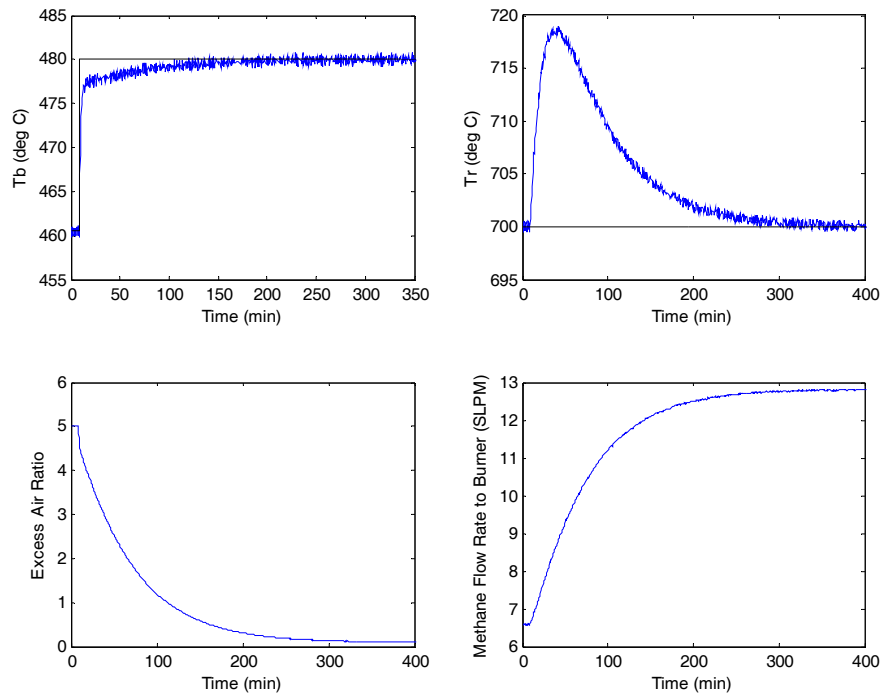
Setpoint changes in both temperatures



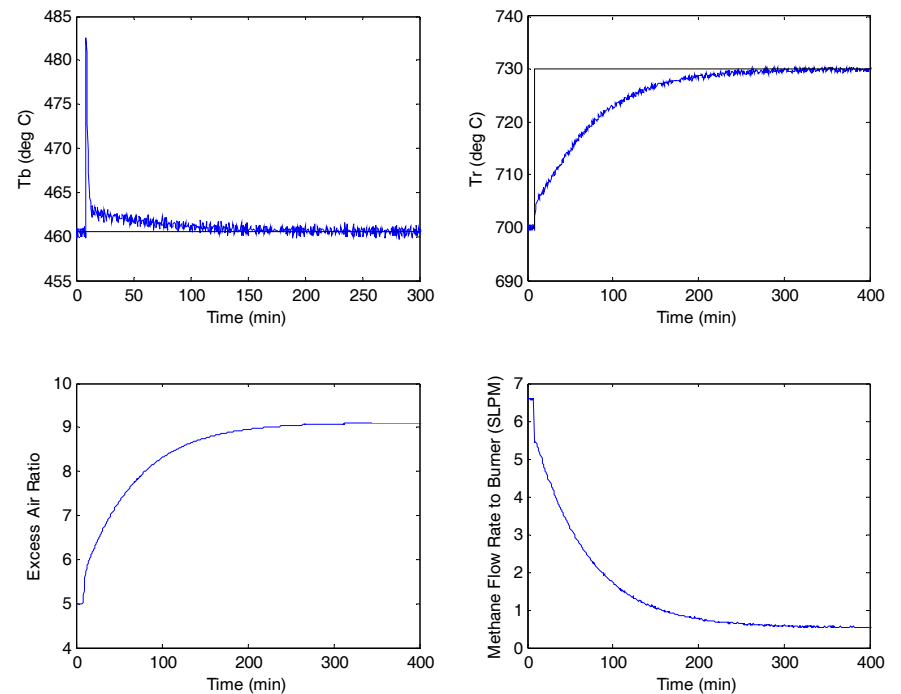
Multivariable Control:

Setpoint changes in only one temperature

Setpoint change in burner temperature



Setpoint change in reformer temperature



Disturbance Rejection:

First-order controller differences

PI controller w/ disturbance rejection
for first order transfer functions

$$g_c = kc \left(1 + \frac{1}{\tau_I s} \right)$$

$$kc = \frac{2\tau_p - \lambda}{kp\lambda}$$

$$\tau_I = \frac{2\tau_p\lambda - \lambda^2}{\tau_p}$$

$$g_c = -0.0383 \left(1 + \frac{1}{692.96s} \right)$$

PI controller w/o disturbance rejection
for first order transfer functions

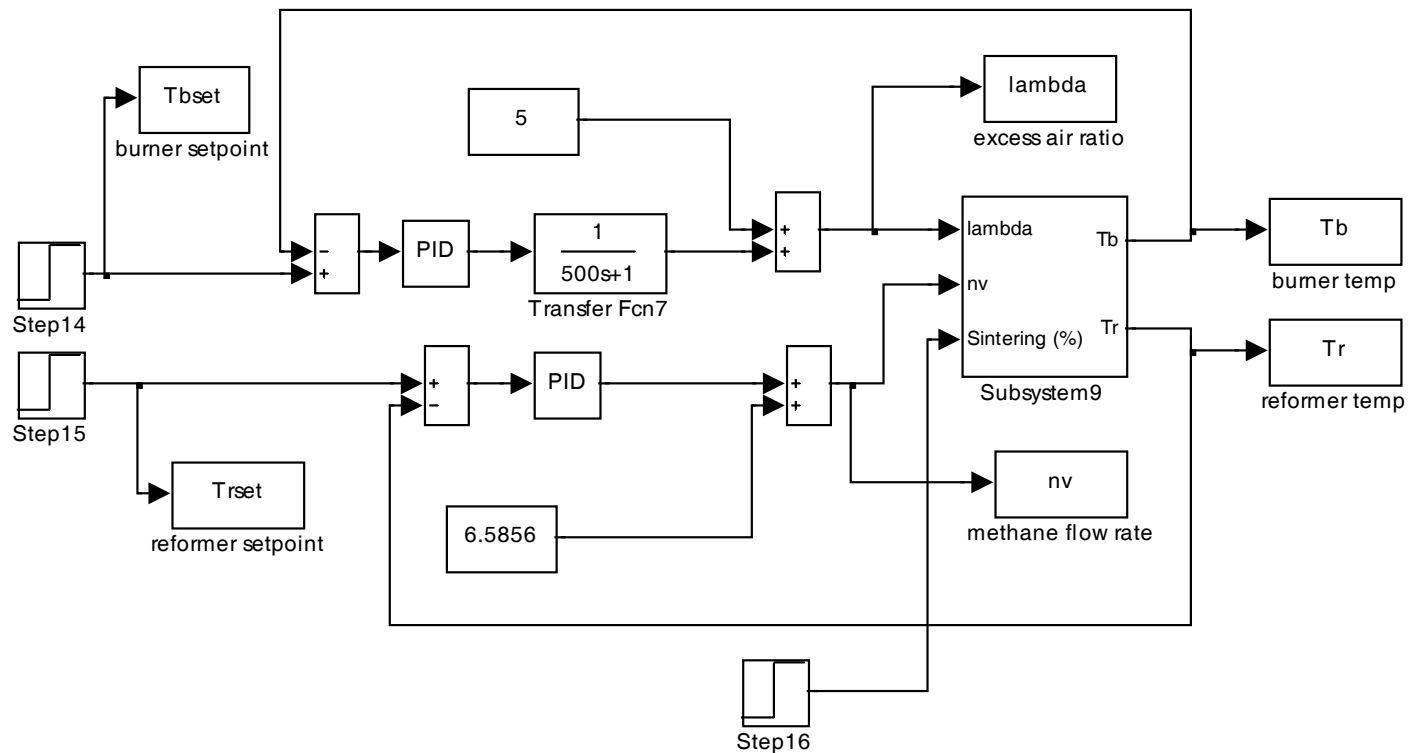
$$g_c = kc \left(1 + \frac{1}{\tau_I s} \right)$$

$$kc = \frac{\tau_p}{kp\lambda}$$

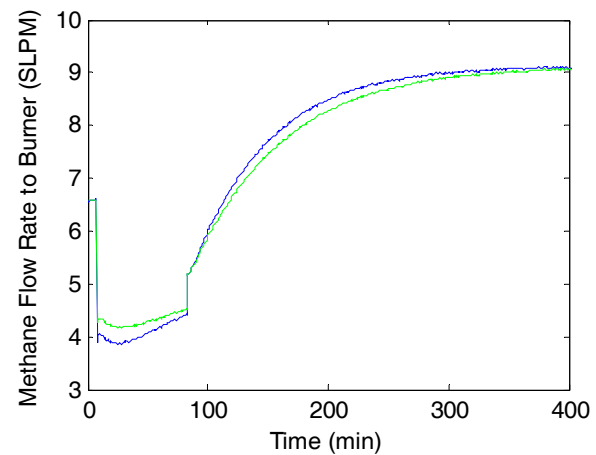
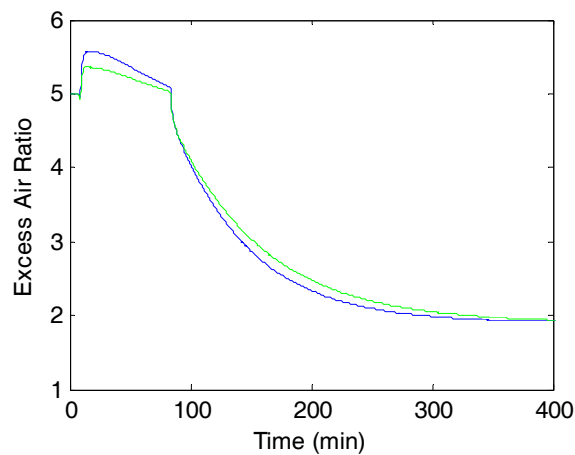
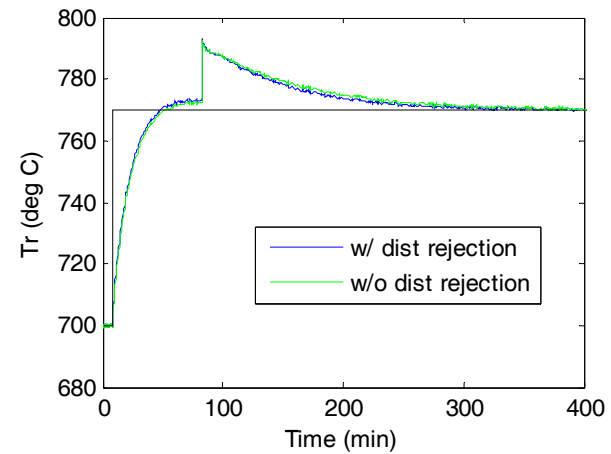
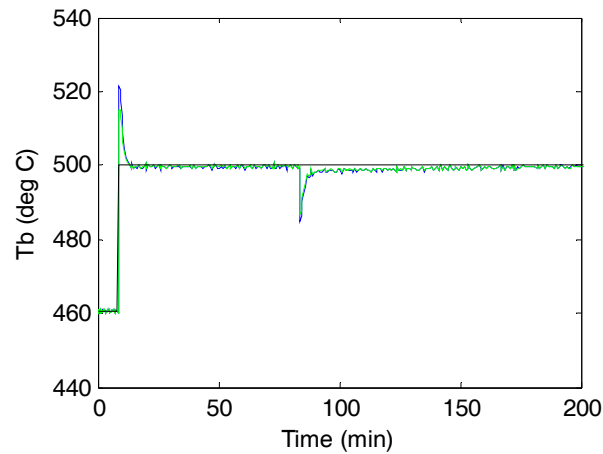
$$\tau_I = \tau_p$$

$$g_c = -0.0332 \left(1 + \frac{1}{710s} \right)$$

Simulink Diagram: System with catalyst sintering disturbance



Disturbance Rejection: 100% Sintering



Directional Sensitivity: Scaling the ranges

	Min Value	Nominal Value	Max Value	½ Range
y 1 (burner)	260.524	460.524	660.524	200
y 2 (reformer)	450.006	700.003	950	249.997
u 1 (excess air)	0	5	10	5
u 2 (methane)	0	6.5846	13.1712	6.5846

Scaled Output Matrix

$$S_o = \begin{bmatrix} \frac{1}{\frac{1}{2} \text{range}(y1)} & 0 \\ 0 & \frac{1}{\frac{1}{2} \text{range}(y2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{200} & 0 \\ 0 & \frac{1}{249.997} \end{bmatrix}$$

Scaled Input Matrix

$$S_I = \begin{bmatrix} \frac{1}{\frac{1}{2} \text{range}(u1)} & 0 \\ 0 & \frac{1}{\frac{1}{2} \text{range}(u2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{6.5846} \end{bmatrix}$$



Directional Sensitivity:

Scaled gain matrix

$$G^* = S_O \times G \times S_I^{-1}$$

$$G^* = \begin{bmatrix} -0.7137 & -0.6385 \\ -0.9101 & -0.9402 \end{bmatrix}$$

Directional Sensitivity: SVD analysis

$$G^* = U\Sigma V^T$$

$$\begin{bmatrix} -0.7137 & -0.6385 \\ -0.9101 & -0.9402 \end{bmatrix} = \begin{bmatrix} -0.5903 & -0.8072 \\ -0.8072 & 0.5903 \end{bmatrix} \begin{bmatrix} 1.6206 & 0 \\ 0 & 0.0555 \end{bmatrix} \begin{bmatrix} 0.7133 & 0.7009 \\ 0.7009 & -0.7133 \end{bmatrix}^T$$

↑
strongest output
direction

↑
weakest output
direction

↑
strongest input
direction

↑
weakest input
direction

Directional Sensitivity: Scaling back to the process

$$y = S_o^{-1} \times y^*$$

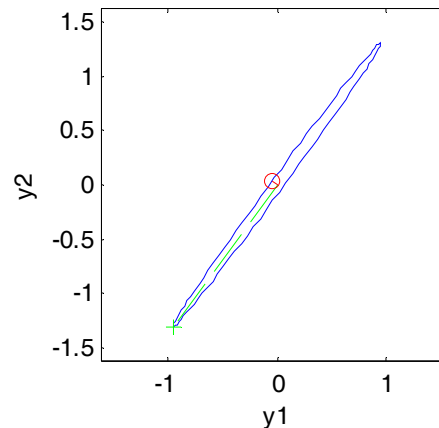
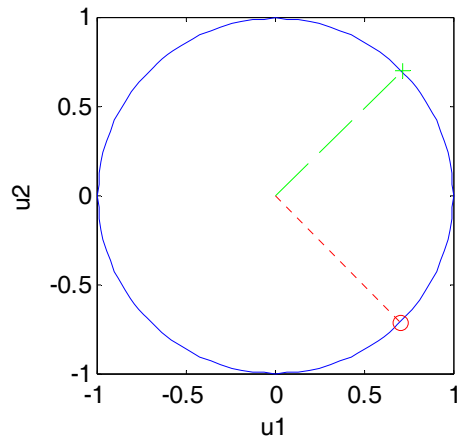
Strong Direction

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -118.06 \\ -201.7976 \end{bmatrix}$$

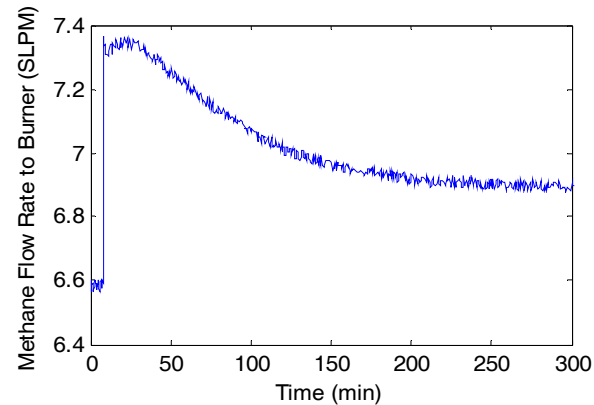
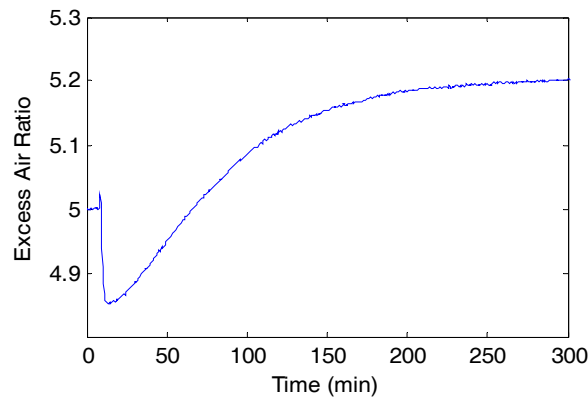
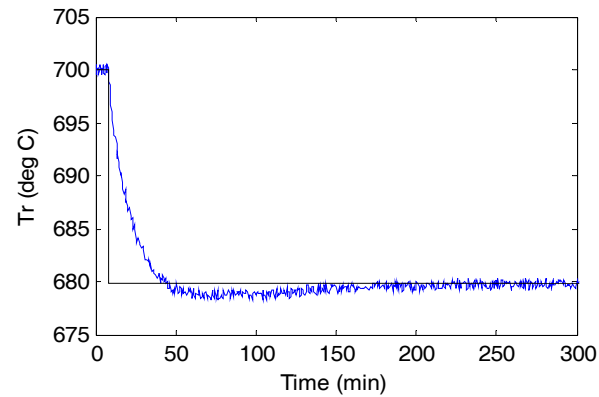
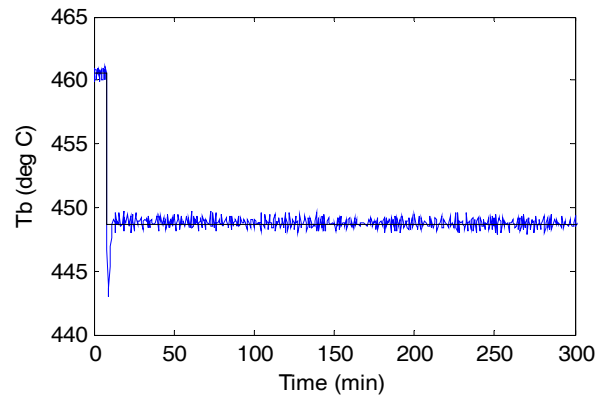
Weak Direction

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -161.44 \\ 147.5732 \end{bmatrix}$$

Input to Output Mapping

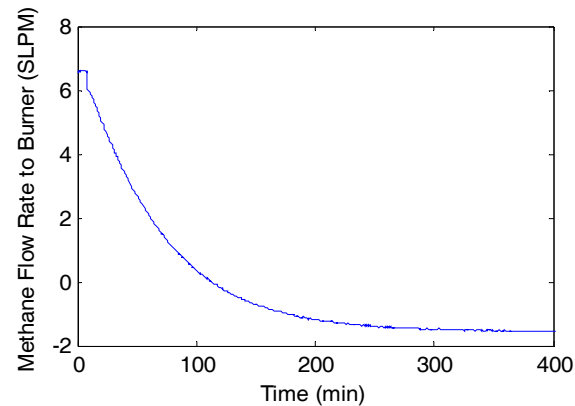
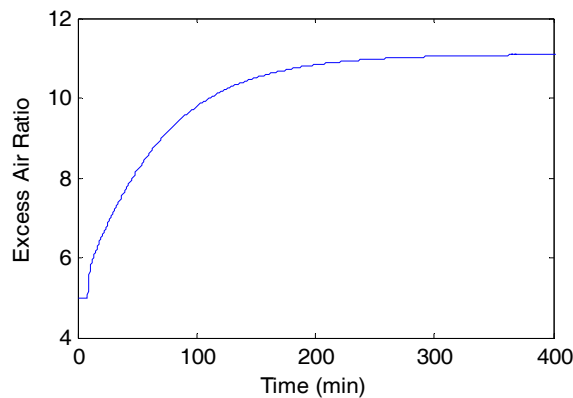
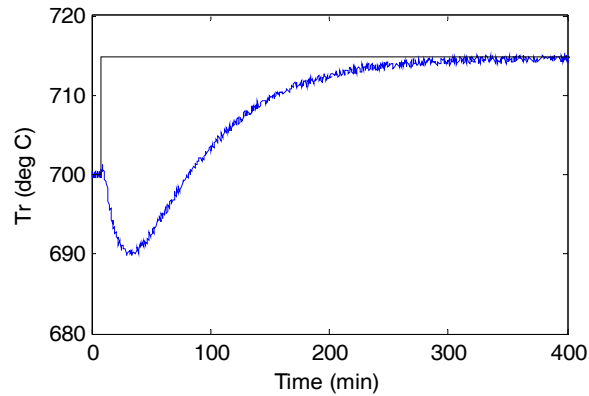
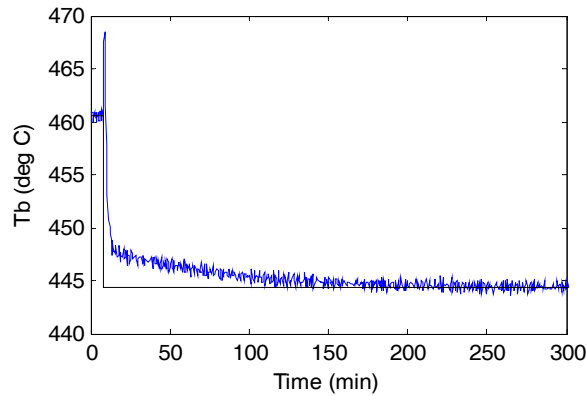


Directional Sensitivity: Changes in the strong direction



Setpoint changes were only 10% of the total

Directional Sensitivity: Changes in the weak direction



Negative
flow rate
←

Setpoint changes were only 10% of total

Conclusion

