Dual-loop SISO controller development for food-extrusion technology:
A case study module

Written and developed by: Joel Schlosburg

Submitted for approval by: Dr. B Wayne Bequette

HOWARD P. ISERMANN DEPARTMENT OF CHEMICAL & BIOLOGICAL ENGINEERING RENSSELAER POLYTECHNIC INSTITUTE TROY, NY 12180

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Abstract

In order to better understand and motivate students to the needs of dynamic modeling and multivariable (MVSISO) design consideration, the case-study of a twin-screw food extruder provides a real-life example with all the requisite components necessary for unit operations management. The study provides a two-input, two-output system able to be simulated via Simulink model that requires knowledge of RGA and SVD analysis. It will also require a student to use extensive knowledge of higher-order systems to model certain responses. All of these needs should create a challenging and engaging system for the students to undertake.

Introduction

The food industry has long been a frontier in cross-disciplinary work of both chemical and biological engineering. The food industry often faces the same issues of design from taking raw materials and cooking/reacting them into a highly-specified, large-scale product. Since the end product is assumedly a consumable food, sterile conditions and proper biological considerations are always a factor in process design. In scale up of large food production plants, there is a greater need for automation to meet such high throughput demands. However, strict constraints on quality control often require constant on-line and off-line testing and measurement.

The screw-based food extruder has been an industrial standard in food processing for several decades due to its ability to mix, cook, compress, transport, and react ingredients simultaneously. The process is generally a single-unit operation that works similarly to a high-speed bioreactor. Feed ingredients are fed down at one end of the machine, where they are mixed by a (single- or twin-) screw that passes the ingredients down the length of the extruder. The extruder can then be carefully regulated for cooking by manipulating the extruder barrel temperature surrounding the mixing ingredients. The speed and temperature of the mixing chamber determines the amount of cooking, moisture, extent of mixing, and exiting temperature of the final product. Upon its exit, an extruder often will have a nozzle to compact, and a die to shape, the exiting product for cutting and further downstream processing and packaging. Currently, single-screw operations are the most common in plant use. However, the twin-screw extruder is the most newly-implemented and most studied because of its greater ability to manipulate individual operating parameters. A typical food extruder setup is shown in figure 1.
Control History

There have been a number of studies performed on the automated control of the twin-screw food extruders. There are three major groups recently working on the various aspects of this control problem that dominate the literature that will be discussed. All use slightly different control techniques and goals, but all use models based on experimental control of producing a cornmeal-based puffed snack. The common theme is to use the experimental data from pilot-scale units to develop transfer functions for controller design.

Dr. Rosana Moreira at Texas A&M (Schonauer 1995, 1996, & 1997) has done a number of studies on extrusion using a color monitoring system over the final product to make assessments of moisture and cooking quality measurement correlations, as well as use of thermocouples and moisture gauges to determine thermal and die dynamics. The ultimate goal was to fully predict both on-line and off-line quality standards which can include: “color of extrudate, bulk density, expansion (diameter, lineal, ratio), texture (breaking strength), water solubility index (WSI), water absorption index (WAI), gelatinization, dextrinization, sensory attributes, dimensional (diameter and length), and surface texture”. The ultimate control systems involved manipulation of screw speed,
feed rate, water rate, and barrel moisture. The models were developed to control motor torque, moisture content, product temperature, and specific mechanical energy. These experiments were then used to model processes for both GPC and MPC control.

Another group out of the University of Newcastle in Australia (Wang 2001, 2004) designed several systems of MPC controllers using screw speed, motor torque, specific mechanical energy, and liquid injection rates as the manipulated variables. They found that die pressure and internal zone temperatures to not have enough direct control on many of the desired output product qualities to make proper manipulated inputs, a finding common to almost all the studies done in this area. Part of the process this group followed in order to develop minimally complex control strategies was a series of transfer function matrices to relate the most relevant inputs and output. However, many of these models were highly complex and involved dynamics way too complicated for our desired modeling basis.

The final investigator of the twin-screw control problem is a Dr. Steven Mulvaney, a food sciences professor at Cornell University (Lu 1993, Singh 1994, and Haley 2000). Almost all the same manipulated inputs and output objectives of many of the other groups were used in the experiments, but with much more focus on developing a wide range of models. Over a series of several experiments, models over a wide range of operating conditions and model order were derived from the collected data. One paper in particular (Singh 1994) focused on low-order models for the use of MIMO PID control, which provided the data and basis for this control case-study.

**Extruder Setup**

The extruder examined in this case-study is a twin-screw extruder with manipulated inputs screw speed (SS) and moisture content (MC). The measured outputs of the system are the product temperature (PT) and the motor torque (MT). The disturbance input on the system is the fluctuation in the jacketed barrel temperature. All temperatures in the system are measured in Celsius, the screw speed is measured in revolutions per minute (rpm), and the motor torque and moisture content is a unitless percentage. The steady-state values, and the constraints on input range, are shown below:
### Table 1 – Values and ranges for inputs and outputs of plant model.

<table>
<thead>
<tr>
<th>Measured Value</th>
<th>Steady-State Value</th>
<th>Range of Manipulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screw Speed</td>
<td>250 rpm</td>
<td>150-350 rpm (±100)</td>
</tr>
<tr>
<td>Moisture Content</td>
<td>18%</td>
<td>13%-23% (±5)</td>
</tr>
<tr>
<td>Barrel Temperature</td>
<td>121°C</td>
<td>101°C-141°C (±20)</td>
</tr>
<tr>
<td>Motor Torque</td>
<td>64.7%</td>
<td></td>
</tr>
<tr>
<td>Product Temperature</td>
<td>161.1°C</td>
<td></td>
</tr>
</tbody>
</table>

Based on transfer functions derived by Singh & Mulvaney (1994), a Simulink model was developed to represent the actual plant behavior. The nominal values and input constraints listed in table 1 were embedded into the system, and random noise was added to the measurement outputs to provide some uncertainty to simulate actual plant outputs. The transfer functions used to represent the plant were:

$$
\begin{bmatrix}
MT \\
PT
\end{bmatrix} =
\begin{bmatrix}
-0.1146 & -0.02627 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.1084 & -0.04034 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0497 & -0.01495 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.03125 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.01198 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00823 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.01455 & -0.006732 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.007813 \\
\end{bmatrix}
\begin{bmatrix}
SS \\
MC \\
BT
\end{bmatrix}
$$

(1)

This transfer function matrix was then transferred into the model using the following state-space values:

$$
A = 
\begin{bmatrix}
-0.1146 & -0.02627 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.1084 & -0.04034 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0497 & -0.01495 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.03125 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.01198 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00823 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.01455 & -0.006732 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.007813 \\
\end{bmatrix}
$$

$$
B = 
\begin{bmatrix}
0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
.0625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & .25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.03125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .0625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

$$
C = 
\begin{bmatrix}
0.06137 & -0.03363 & 0 & 0 & -0.4042 & -0.105 & 0 & -0.0316 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.03873 & 0 & 0 & -0.2295 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.05062 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

(2)
Model Development

The first step in plant controller design is the step test, and modeling of the system dynamics. The models were developed strictly from empirical observation of step increases in both manipulated inputs, while maintaining all other inputs at steady state. The gain was determined by the change in the output steady-state value divided by the magnitude of the step change \( k_r = \frac{\Delta y}{\Delta = \frac{\Delta y}{\Delta}} \). The initial time constants were estimated using the knowledge of a Laplace function, which is based on natural functions, yielding the following equation:

\[
y(t = \tau_p) = \Delta y(1 - e^{-\frac{t}{\tau}}) = 0.632\Delta y
\]

Once the final output change was determined, this change was multiplied by .632, and the time at which this value change occurred was the initial time constant. Time delays were also observed and subtracted from this time, along with the time until the step change. This allowed the accurate modeling of most of the desired transfer functions, but there are two functions in equation 1 that have numerator dynamics not able to be modeled directly by first-order systems. The first is the SS-MT function, which has noticeable inverse response and second-order dynamics. In order to properly model the system, a primary assumption to make is that the system is critically damped \((\zeta=1)\). From figure 3-9 in the Bequette (2003), one can find \( \tau \) by measuring a characteristic percentage of the rise to steady-state, and find the initial \( \tau \) for the system. So the time to .75 of the final \( \Delta y \) is 58 seconds. This number can then be divided by the \( t/\tau \) value of 2.5, based on the chart, but only after the numerator time constant is factored in. Based on the shape of the curves in figure 3-11, and some guess-and-check simulation, it is found that the proper system is around a \( \tau_n = -15 \) seconds, which causes the time constant equation to be:

\[
\tau_{1,2} = \frac{(58-15)/2.5}{2.5} = 17.2 \text{ seconds}
\]

The second function to be determined has positive numerator dynamics, which causes an apparent overshoot of the steady-state value. Since the controllers being designed should be fairly competent at preventing excessive overshoot with proper tuning, it was decided to still model this system as a first-order plus time delay model. However, this could be modeled much the same way discussed above, which would introduce its own error, as this function is not critically damped. However, the same techniques using both figures 3-9 and 3-11 can yield reasonable results. The resulting transfer function matrix developed strictly from empirical observation is:
Using these values, an overlay of the actual plant behavior versus the empirically modeled behavior can be compared for accuracy. These overlays are shown in figures 2 and 3:

**Figure 2 -** Comparison of step response increase in screw speed of the empirical models versus the actual plant.
Figure 3 - Comparison of step response increase in moisture content of the empirical models versus the actual plant.

Single Loop Control Systems
In developing a set of four unique SISO loops, the transfer functions of the loops were used to create Internal Model Control (IMC) based-Proportional Integral Derivative (PID) controllers. Due to the constraints on both manipulated inputs, and the saturation limits in place, it optimal setup is to include an antireset windup (ARW) function in the controllers. These will prevent the controllers from continuing to ask for more input action when these saturation limits are encountered. This is crucial, as the action of reset windup can delay corrective action due to long periods at or near constraints. The PID control tuning parameters were derived based on function tables for IMC based-PID control, using the observed parameters of the transfer functions in the model (Bequette 2003):
### Table 2 – Closed-Loop PID tuning parameter values and λ ranges

<table>
<thead>
<tr>
<th>Loop name (input-output)</th>
<th>$k_c$</th>
<th>$\tau_i$ (s)</th>
<th>$\tau_d$ (s)</th>
<th>Optimal Experimental λ Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS-MT</td>
<td>$\frac{-107.5}{\lambda + 15}$</td>
<td>34.4</td>
<td>8.55</td>
<td>30-40 sec</td>
</tr>
<tr>
<td>SS-PT</td>
<td>$\frac{455.17}{\lambda}$</td>
<td>54.62</td>
<td>0</td>
<td>25-35 sec</td>
</tr>
<tr>
<td>MC-MT</td>
<td>$\frac{-54.474}{\lambda + 19.5}$</td>
<td>31.05</td>
<td>7.25</td>
<td>100-110 sec</td>
</tr>
<tr>
<td>MC-PT</td>
<td>$\frac{-43.54}{\lambda + 20.5}$</td>
<td>104.5</td>
<td>16.48</td>
<td>30-40 sec</td>
</tr>
</tbody>
</table>

Upon implementing the controllers, a controller parameter (λ) is used to adjust the robustness and accuracy of the control action. An optimal λ range for each control loop is listed in Table 2 above. These are not absolute limits, but they are considered the best balanced for speed and accuracy in simulation experiments. These optimal lambda values are apparent in figures 4 and 5, which show the closed-loop response of the four independent loops to a step change. Our next step was to determine which inputs should have primary control of certain outputs.

[Figure 4 – Closed loop response to setpoint increases in MT (left) and PT (right) by manipulation of SS.]
Relative Gain Analysis and MVSISO Pairing

The relative gain is the ratio of the gain of a particular input-output relationship with the loops open and the loops closed. With this current 2x2 system, the RGA is calculated as:

\[
\Lambda = \begin{bmatrix}
\lambda_{11} & \lambda_{12} \\
\lambda_{21} & \lambda_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{k_{i1}k_{22} - k_{i2}k_{21}}{k_{i1}k_{22} - k_{i2}k_{21}} & \frac{-k_{i1}k_{22} - k_{i2}k_{21}}{k_{i1}k_{22} - k_{i2}k_{21}} \\
\frac{k_{i1}k_{22} - k_{i2}k_{21}}{k_{i1}k_{22} - k_{i2}k_{21}} & \frac{k_{i1}k_{22} + k_{i2}k_{21}}{k_{i1}k_{22} - k_{i2}k_{21}}
\end{bmatrix} = \begin{bmatrix}
.88 & .12 \\
.12 & .88
\end{bmatrix}
\]

(6)

From these values, it can be concluded that the best pairing will be the screw speed controlling the motor torque, and the moisture content controlling the product temperature. This is because the closer the relative gain value chosen is to one, the more likely the loop will be able to be reasonably controlled in cases of any paired control loops' failure. As the loop opens up, a value close to one means that the system should be relatively unchanged, and the adaptability of the IMC-PID controllers should be sufficient to maintain desired setpoints. With the values of the chosen relative gains being between 0 and 1, the controller gains will be most effective if detuned. This will prevent any instability in the case of loop failure, as the relative gain suggests this occurrence may cause the controller to be overaggressive. The detuning of the controller gains is done by multiplying the \( k_c \) by the chosen value of \( \lambda \).

It should be noted that in the paper this case study is based on, barrel temperature was used as a manipulated input as opposed to an input disturbance. Singh (1994) found that using barrel temperature as opposed to moisture content had the
lowest level of interaction between loops. While this provides for optimal control, as a learning experience it is relatively minimal, and it reduces the insight necessary in choosing a proper RGA pairing, and the necessity to explore the other possible control designs. This is due to this other option resulting in negative RGA values, leaving only one pairing choice.

**Singular Value Decomposition (SVD) & MVSISO Performance**

The SVD allows for the determination of the strongest and weakest setpoint directional changes to be made, and gives insight into the flexibility of the MVSISO system to make a full range of setpoint changes, allowing for minimal operator knowledge of the system for ease of operation. The goal is to first scale the gain matrix of the transfer function 2x2 system by the input and output ranges, and then determine the SVD based on this scaled matrix. Once determining the strongest and weakest setpoint step changes possible, the output values in the SVD should be scaled back to the original range of the outputs, and implemented. If the system is truly flexible and capable of a variety of controller moves, then the performance in both set of directions should be stable and timely.

The scaled gain matrix for this system is determined in equation 7, and the SVD resulting from this scaled matrix is shown as equation 8.

\[
G^* = S_0 \cdot G \cdot S_1^{-1} = \begin{bmatrix}
1/16.3 & 0 \\
0 & 1/12.5
\end{bmatrix} \begin{bmatrix}
-.32 & -.87 \\
100 & 0
\end{bmatrix} = \begin{bmatrix}
-.1963 & -.267 \\
.960 & -.960
\end{bmatrix}
\]  

\[
G = USV^T = \begin{bmatrix}
-.8771 & .4803 & 2.1948 & 0 \\
.4803 & .8771 & 0 & .9754 \\
2.1948 & 0 & .9946 & -.1034 \\
0 & .9754 & -.1034 & -.9946
\end{bmatrix}
\]  

\[
\sigma_{\text{max}} / \sigma_{\text{min}} = 2.25. \text{ While the most ideal control system would have a condition number close to one, very few actual operations are close to this number, and values between 2 to 3 are fairly “well-conditioned”. Values of around 10 or greater is when it is likely there are certain directional changes the developed system simply cannot make, limiting the ability of the controller operator. The output changes for the strongest (MT decrease, PT increase) and weakest (MT increase, PT decrease) directions to demonstrate the system’s paired performance are calculated as:}
\[ Y^* = 0.25 \cdot S^{-1}_o \cdot U = 0.25 \cdot \begin{bmatrix} 16.3 & 0 \\ 0 & 12.5 \end{bmatrix} \cdot \begin{bmatrix} -0.8771 & 0.4803 \\ 0.4803 & 0.8771 \end{bmatrix} \cdot \begin{bmatrix} -3.60 \\ 1.50 \end{bmatrix} \quad (9) \]

The control responses to these output setpoint changes are shown in figures 6 and 7:

**Figure 6** – MVSISO response of the strong RGA pairing in the directions determined the strongest by the SVD analysis.

**Figure 7** – MVSISO response of the strong RGA pairing in the directions determined the weakest by the SVD analysis.
Figures 6 and 7 demonstrate that in both the strongest and weakest directions, stable changes can be made simultaneously to both output setpoints. Both changes reach steady-state target values at approximately the same time. All changes appear to have slight issues of overshoot, which can be minimized by changing both setpoints in certain ratios as to allow the interaction between control loops to be minimized. In this case, the product temperature change was about half that of the motor torque change. Noticeably less overshoot is seen when both are changed at roughly equal value. To verify and the original RGA pairings made, figure 8 shows the performance of the opposite pairing in the supposed strongest SVD setpoint directions. Notice that even the strongest change results in oscillatory and unstable conditions.

**Figure 8** – The improper RGA pairing performance due to strong SVD setpoint changes.

**Disturbance Rejection**

One of the keys to proper daily operation is not changing setpoints, but maintaining setpoints at nominal levels. Operators often have hundreds to thousands of loops in a plant to look after, and rarely is a given loop being changed on a regular basis. However, flowrate and upstream product variations are amongst many causes to why many of the values assumed to be constant in model and controller development are not so. These values can fluctuate up and down constantly, much the way the noise on the
measured outputs do. In order to test the ability of the designed controller to handle daily issues of disturbance rejection, the barrel jacket that surrounds the extruder will be increased in temperature to see how well the MVSISO system can maintain the output at their desired setpoints. Figure 9 shows the controllers ability to withstand a 10°C increase in temperature, beginning at 10 seconds, and taking full effect around 200 seconds. Both values quickly return to their nominal values, and without varying much further outside the range of measurement noise.

![Disturbance Rejection Plot](image)

**Figure 9 – Disturbance rejection of a 10°C increase in barrel jacket temperature.**

**Further Study**

While this case-study already requires the student to use a bit more modeling and dynamic process knowledge than many current systems, this system can be made even more difficult by forcing the modeling of both the positive and negative numerator dynamic models. However, it is a valuable lesson for students to see that even a system they seemingly can’t model accurately can be reasonably controlled by a properly tuned IMC system. The fact that the dynamics were not exactly modeling the plant did allow for some overshoot and need for high lambda values, but the resulting control system was still flexible and relatively efficient. One issue that will be unavoidable within this
system is the existence a right-half-plane transmission zero in the current 2x2 matrix, which will slow the setpoint tracking in the closed loop system.

\[
tzeros = \begin{bmatrix}
0.1282 \\
-0.0128 \\
-0.0194 \\
-0.0459 \\
-0.0703
\end{bmatrix}
\]

One positive possible effect of performing the positive numerator dynamics modeling also force the student to explore deriving their own IMC-PID parameters, as a numerator dynamic plus time delay system is not found within table 9-2 (Bequette 2003). This can only be done with substantial advisor interaction, as the system is not even critically damped, so some guidance would likely be needed to begin modeling.

References

Supplemental Figures:

Case-Study Plant Model for the Twin-Screw Food Extruder

Subsystem of "Actual Plant" Unit Based on State-Space Model
Empirically-Based User Built Model of Extruder Plant Dynamics

Typical SISO Control Loop (this loop representing the SS-MT loop)