Locally Divergence-Free Discontinuous Galerkin Methods for the Maxwell Equations

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Abstract

In this paper, we develop the locally divergence-free discontinuous Galerkin method for numerically solving the Maxwell equations. The distinctive feature of the method is the use of approximate solutions that are exactly divergence-free inside each element. As a consequence, this method has a smaller computational cost than the one of the discontinuous Galerkin method with standard piecewise polynomial spaces. We show that, in spite of this fact, it produces approximations of the same accuracy. We also show that this method is more efficient than the discontinuous Galerkin method using globally divergence-free piecewise polynomial bases. Finally, a post-processing technique is used to recover $(2k+1)$-th order of accuracy when piecewise polynomials of degree k are used.