Final 3 December 20, 2004

Probability Theory and Applications

Name:_____

Present your work **neatly** in the working space provided. For additional space use back of the page. Mark each part and circle your answer whenever possible. Answers should be in fractional or decimal form unless otherwise indicated. **Remember to specify all necessary bounds on the variables.**

Normal tables are attached.

Problem No.	**Points**	
1	20	
2	20	
3	20	
4	20	
5	20	
extra credit	6	
Total	100	

1. (20 pts) N patients taking a drug are tested to see if they have a side effect. We expect 25% of the patients to have the side effect. Assume all of the patients are independent. Let X be the number of patients of the N experiencing the side effect.

(a) (6 pts) For $N = 15$, calculate $P(X \geq 5)$.

(b) (14 pts) For $N = 120$, use the normal approximation to the binomial to determine $P(X = 32)$.

2. (20 pts) Let X be a continuous random variable with pdf given by

$$f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases} \tag{1}$$

(a) (6 pts) Calculate $P(X < 0.5 | X \geq 0.25)$.

(b) (14 pts) Determine the pdf of $Y = e^X$. Don't forget the value set of Y.

3. (20 pts) You plan to sell raffle tickets for a student club fund raiser in the Student Union. From past experience, you know approximately 10% of the people will buy the ticket when you ask. Assume the decision of each person is independent.

(a) (6 pts) On Tuesday you will keep asking people in the union to buy raffle tickets until you sell 10 tickets. Let X be the number of people you have to ask until you sell the 10 ticket on Tuesday. What named distribution would be the best choice for the distribution of X? Make sure you specify it's parameters.

(b) (14 pts) On Wednesday you will keep asking people in the union to buy raffle tickets until you sell 20 tickets. Let Y the number of people you have to ask until you sell the 20 tickets on Wednesday. Use the m.g.f method to determine the distribution of $W = X + Y$, the total number of people you ask to by tickets over the two days. Don't forget to specify the value set.

4. (20 pts) You roll three dice. Assume the dice are fair dice with 6 equally likely sides labelled 1 to 6, and that the rolls are independent. The value of the roll is the number on the die that is uppermost.

(a) (12 pts) We want to know what the probability is that all three rolls have values that are less than some value m? So for example:

$$P(M \leq 1) = P(\text{first roll} \leq 1, \text{second roll} \leq 1, \text{third roll} \leq 1).$$

Complete the following table:

$P(M \leq 1)$	$=$
$P(M \leq 2)$	$=$
$P(M \leq 3)$	$=$
$P(M \leq 4)$	$=$
$P(M \leq 5)$	$=$
$P(M \leq 6)$	$=$

(b) (8 pts) Use your answer in part a, to construct a table representing the p.d.f of M.

5. (20 pts) You are in charge of quality control of chocolate candies. You pick a random sample of 16 candies and weigh each one. Let X_i be the weight of the i^{th} candy and let $\bar{X}$ be the sample mean (the sum of all the candy weights divided by 16). Assume that the weights of the candies are independent and identically distributed, but you don't know their distribution other than the mean and variance. The mean of X_i is $\mu = 3$ and the standard deviation of X_i is 0.04.

(a) (6 pts) What it the mean and variance of $\bar{X}$?

(a) (14 pts) You are not sure that you have taken a big enough sample. You want the error between the observed mean and the sample mean to be less than 0.05, Is your sample size okay? What is the probability $P(|\bar{X} - \mu|) > .05$? If you make additional assumptions, indicate what they are and why they are valid.

Extra Credit (8 pts) Let X and Y be continuous random variables with joint pdf given by

$$f_{X,Y}(x,y) = \begin{cases} e^{-2x-3y} & 0 < Y \quad 0 < X \\ 0 & otherwise \end{cases} \tag{2}$$

Determine the pdf of $U = \frac{X}{Y}$.