Homework 7

Probability Theory and Applications Statistics

Assignment due 12/6 in class (if you have exam that day, you can turn in my box up to Wednesday at 3 when solutions will be posted.

Functions of Random Variables and CLT

Turn in the starred (*) problems.

1. *Suppose that a random variable X can have each of the seven values -3, -2, -1, 0, 1, 2, 3 with equal probability. Determine the p.d.f. of $Y = X^2 - X$.

2. *Suppose X has the pdf $f(x) = (3/8)x^2,\ 0 < x < 2$. Find the pdf of $Y = -X^2$.

3. *The random variables X and Y are independent and their moment generating functions (mgf's) are: $M_X(t) = (1 - \frac{t}{2})^{-3}$ and $M_Y(t) = (1 - \frac{t}{3})^{-4}$. What's the distribution of Y? Obtain the mgf of $U = 4X + 6Y$, identify the distribution of U and find the mean and variance of U.

4. * Let X and Y be independent random variables with respective pdf's $f(x) = 3x^2,\ 0 < x < 1$ and $g(y) = 4y^3,\ 0 < y < 1$. What's the pdf of $Z = \frac{Y}{X}$. Find $E(\frac{Y}{X})$.

5. The pdf of X is $f(x) = 4x^3,\ \ 0 < x < 1$. Find the pdf of $(x - \frac{1}{2})^2$

6. Let (X, Y) have the joint pdf $f(x, y) = e^{-x},\ 0 < x, < \infty,\ 0 < y < 1$. If $X = Z + 2Y$, what is the joint pdf of X and Z?

7. The random variables X_1 and X_2 Have the joint pdf $f(x_1, x_2) = x_1 + x_2,\ \ 0 < x_1 < 1,\ \ 0 < x_2 < 1$. Find the pdf of $U = X_1 + X_2$.

8. In a newspaper ad, a car dealer lists a 1986 Chrysler Le Baron, a 1985 Ford Escort, and a 1987 Buick Skylark. If the numbers of inquiries he will get about these cars may be regarded as independent random variables having Poisson distributions with the parameters $\lambda_1 = 3.6$, $\lambda_2 = 5.8$, and $\lambda_3 = 4.6$, what is the probabilities that he will receive at least 18 inquires about the cars?

9. * Suppose a random sample of size 20 is selected from a lot of candy canes. If 10% of the candy canes are defective, find the probability that the sample will contain at least two defectives by the following methods:

   (a) Using the normal approximation.

   (b) Using the exact binomial tables.

10. (* all) A random sample of size n is to be taken from a population with mean μ and standard deviation σ. For each case, determine, the required sample size n.

    (a) We want $P(|\bar{x} - \mu| \leq \frac{\sigma}{2})$ to be at least .95.

(c) We want $P(|\bar{x} - \mu| \leq \frac{\sigma}{5})$ to be at least .99.

11. The random variables X_1 and X_2 Have the joint pdf $f(x_1, x_2) = x_1 + x_2, \quad 0 < x_1 < 1, \quad 0 < x_2 < 1$. Find the pdf of $U = X + Y$.

12. * Suppose that $X_1, X_2, \ldots, X_n$ are independent and identically distributed with the common pdf $f(x) = 2x, \; 0 < x < 1$. Let $V = max\{X_1, X_2, \ldots, X_n\}$. Find the pdf of V, and calculate E(V) when $n = 10$.