

Probability Theory and Mathematical Statistics I
Homework 5 - Multivariate Distributions

The starred problems (*) are assigned. The rest are practice. Assignment is due in class on Tuesday November 1. No late assignments.

1. If the annual proportion of erroneous income tax returns filed with the I.R.S. can be looked upon as a random variable having a beta distribution with $\alpha = 2$ and $\beta = 9$, what is the probability that in any given year there will be fewer than 10 percent erroneous returns?
2. * If the annual proportion of new restaurants that fail in a given city may be looked upon as a random variable having a beta distribution with $\alpha = 1$ and $\beta = 4$, find
 - (a) the mean of this distribution, namely, the annual proportion of new restaurants that can be expected fail in the given city;
 - (b) the probability that at least 25 percent of all new restaurants will fail in the given city in any one year.
3. * Suppose X has the moment generating function (mgf) $M(t) = \frac{1}{4}(3e^t + e^{-t})$, find the mean and variance of X .
4. * Two textbooks are selected at random from a shelf that contains 3 statistics texts, 2 mathematics texts, and 3 physics texts. If X is the number of statistics texts and Y is the number of Mathematics texts selected, construct a table showing the values of the joint probability distribution of X and Y . Find
 - (a) the marginal distribution of X ;
 - (b) the marginal distribution of Y ;
 - (c) the conditional distribution of X given $Y = 1$;
 - (d) the conditional distribution of Y given $X = 1$;
5. * (b only) Suppose X has the pdf $f(x) = 3x^2$ for $0 < x < 1$, $f(x) = 0$ otherwise, and the conditional pdf of Y given $X = x$ is

$$f(y|x) = \begin{cases} \frac{3y^2}{x^3} & \text{for } 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Determine:

- (a) the marginal pdf of Y
- (b) the conditional pdf of X given $Y = y$.

6. * If P , the price of a certain commodity (in dollars), and S , total sales (in 10,000) units, are random variables whose joint distribution function can be approximated with the joint probability density

$$f(p, s) = \begin{cases} 5pe^{-ps} & \text{for } 0.20 < p < 0.40, s > 0 \\ 0 & \text{otherwise} \end{cases}$$

find the probabilities that

- the price will be less than 30 cents and sales will exceed 20,000 units;
- the conditional density function of X give $P = p$;
- the probability that sales will be less than 30,000 units given $P = 25$ cents.
- the probability that sales will be less than 30,000 units given $P \leq 25$ cents.
- the expected sales when $P = .25$.

7. Suppose X and Y have the joint pdf

$$f(x, y) = \begin{cases} \frac{3}{16}(4 - 2x - y) & \text{for } 0 < x, 0 < y \text{ and } 2x + y < 4 \\ 0 & \text{otherwise} \end{cases}$$

- Find the conditional pdf of Y given $X = x$
- $P(Y \geq 2 | x = 0.5)$.

8. A marksman is aiming at a circular target of radius 1. If we draw a rectangular system of coordinates with its origin at the center of the target, the coordinates of the point of impact, (X, Y) , are random variables having the joint probability density

$$f(x, y) = \begin{cases} \frac{1}{\pi} & \text{for } 0 < x^2 + y^2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine:

- $P[(X, Y) \in A]$, where A is the sector of the circle in the first quadrant between the radii along the line $y = 0$ and $y = x$; the marginal pdf of Y
- $P[(X, Y) \in B]$, where $B = \{(x, y) | 0 < x^2 + y^2 < \frac{1}{2}\}$.