

Probability Theory and Mathematical Statistics I
Homework 3 - Discrete Distributions

The starred problems (*) are assigned. The rest are practice. Assignment is due in class on Friday September 30. No late assignments.

1. Suppose that a random variable X has a discrete distribution with the following p.f.:

$$f(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of the constant c .

2. * A bag contains three coins, one of which has a head on both sides while the other two coins are normal. A coin is chosen at random from the bag and tossed three times. The number of heads is a random variable, say X .
 - (a) Find the discrete pdf of X .
 - (b) Sketch the discrete pdf and the CDF of X .
3. * A discrete random variable has a pdf of the form $f(x) = c(8 - x)$ for $x = 0, 1, 2, 3, 4, 5$, and zero otherwise.
 - (a) Find the constant c .
 - (b) Find the CDF, $F(x)$.
 - (c) Find $P(X > 2)$.
4. A multiple-choice test consists of eight questions and three answers to each question (of which one is correct). If a student answers each question by rolling a balance die and checking the first answer if he gets a 1 or 2, the second answer if he gets a 3 or 4, and the third answer if he gets a 5 or 6, what is the probability that he will get exactly four correct answers? The number of correct answers is a random variable Y . What is the pdf of Y .
5. A social scientist claims that only 50 percent of all high school seniors capable of doing college work actually go to college. Assuming that this claim is true, use the Binomial Table to find the probabilities that among 15 high school seniors capable of doing college work
 - (a) exactly 10 will go to college;
 - (b) at least 10 will go to college;
 - (c) * at most 8 will go to college.
6. * Among the 300 employees of a company, 240 are union members, whereas the others are not. If six of the employees are chosen by lot to serve on a committee that administers the pension fund, find the probability that four of the six will be union members using
 - (a) the formula for the hypergeometric distribution;
 - (b) the binomial distribution as an approximation

7. Peter thinks that Laurie can sink a shot from the foul line at most 5% of the time. They do an experiment, and Laurie manages to sink 6 out of 30. Will Peter admit that he is wrong?
8. Phone calls are received at a certain residence as a Poisson process (e.g. according to a Poisson distribution) with parameter $\lambda = 2$ per hour.
 - (a) * If Billy takes a 10 minute shower, what is the probability that the phone rings during that time?
 - (b) * How long can his shower be if he wishes the probability of receiving no phone calls to be at most .5?
9. The average number of trucks arriving on any one day at a truck depot in a certain city is known to be 12. What is the probability that on a give day fewer that nine trucks will arrive at the depot?
10. * The probability that a certain baseball player gets a hit is 0.3, and we assume that times at bat are independent. What is the probability that he will require five times at bat to get his first hit? Given that he has been at bat 10 times without a hit, what is the probability that it will require five more times at bat for him to get his first hit?