PROBABILITY THEORY AND APPLICATIONS
MATP 4600 and DSES 4750
FALL 2005
LECTURE 2
Sept. 2, 2005

LOOKING BACK (PROBABILITY MODEL) We look back at the the urn problem of Lecture 1 and crystallize it probability model.

A *sample space*, S, is a non-empty set; it models all possible outcomes of an experiment.

> All possible outcomes of drawing two balls simultaneously from an urn that contains 14 numbered balls is modeled by the set
>
> $$S = \{\{i, j\} : 1 \leq i, j \leq 14,\ i \neq j\}.$$

An *event*, E, is a subset of S (an element of 2^S); it models a particular set of outcomes of an experiment.

> The balls in the urn, numbered 1-11 (the red balls) are modeled by the event
>
> $$E = \{\{i, j\} : 1 \leq i, j \leq 11,\ i \neq j\}.$$

A *probability function*, P, is a function defined by the axioms below. It assigns a number, $P(E)$ (between 0 and 1) to each event, E. In so doing it models how likely certain outcomes are to occur: any one of those outcomes, modeled by E.

> For $E = \{\{i, j\} : 1 \leq i, j \leq 11,\ i \neq j\}$, we found that $P(E) = \frac{55}{91}$.

Recall that a probability function is a real-valued function P defined on 2^S such that

- $P(E) \geq 0, \forall E \in 2^S$,
- $P(S) = 1$,

- $P(E_1 \cup E_2) = P(E_1) + P(E_2)$, $\forall E_1, E_2 \in 2^S$, $E_1 \cap E_2 = \emptyset$.

On frequently finds, in the literature, reference to a probability space. A *probability space* is a pair (S, P) were S denotes an abstract non-empty set and $P : 2^S \to \mathbb{R}$ and satisfies the axioms found in the three bullets above. One notes that the notion of a probability space exists independently of any experiment; it is an abstract mathematical construct. (It is frequently studied as such.)[1]

A probability function has several properties that are frequently used in probability modeling. Rather than list them here—they are stated and proved in the text—we introduce them in the examples of Worksheets #1 and #2.

[1]Because our attention is focused on finite and countably infinite sample spaces we can assume that P is defined on all of 2^S. If S is uncountable, it is usually necessary to extract from 2^S a subset, $\mathcal{A}$, called a sigma algebra of sets, and define P on $\mathcal{A}$. In this case a probability space becomes the triple $(S, \mathcal{A}, P)$. However, this idea gets in the way of our initial investigation of probability functions defined on finite and countably infinite sets. So we think of a probability space as the pair (S, P).