Take home make-up   Computational Optimization Spring 2008
Due Tuesday April 15 in class

1. Construct a quadratic penalty function algorithm (N.W. Framework 17.1 page 501) for solving the problem

$$\min f(x) = x_1 \text{ subject to } h_1(x) = x_1^2 + x_2^2 + x_3^2 - 4 = 0$$
$$h_2(x) = x_1 + x_2 + x_3 - 1 = 0$$

Use $P(x,\mu) = f(x) + \frac{\mu}{2}\left(h_1(x)^2 + h_2(x)^2\right)$ and at each iteration use the Exact Newton code used in the takehome to solve the unconstrained problem to a tolerance of 1e-6. Remember from class discussion that we can calculate the Hessian of *P* using the fact that

$$\nabla^2\left(\tfrac{1}{2}h(x)^2\right) = h(x)\nabla^2 h(x) + \nabla h(x)\nabla h(x)^T.$$

In Framework 17.1 pg 501, you need to specify μ_k and τ_k. As noted above τ_k=1e-6 for each iteration. Use μ_0 =1 to start and then double the value of μ_k at each iteration ($\mu_{k+1} = 2\mu_k$). Use x=[0 0 0]' as a starting point.

In your algorithm use the equation

$$\lambda_i = h_i(x_k)\mu_k$$

to update the estimate for each Lagrange multiplier at iteration *k*. Have your output display the values of $x_1, x_2, \lambda_1, \lambda_2$ and $h(x)$ for each iteration. Stop when the change in x is less than or equal 1e-6.

2. Solve the above problem using fmincon. How do your results compare? Use the KKT conditions to determine if you found an optimal solution or not.

Hand in your take home midterm, all the code you wrote for this makeup, a diary of your output, and a discussion of your results from 1 and 2.